AD-782 857

DESIGN AND DEVELOPMENT TESTING OF FREE PLANET TRANSMISSION CONCEPT

Neil A. De Bruyne

Curtiss Wright Corporation

Prepared for:

Army Air Mobility Research and Development Laboratory

April 1974

EUSTIS DIRECTORATE POSITION STATEMENT

This report was the initial effort to demonstrate and evaluate the free planet transmission concept for use in a helicopter drive train system. The limited test results indicate that the free planet transmission concept is feasible and competitive with existing gearboxes.

This report has been reviewed by this Directorate and is considered to be technically sound. The technical monitor for this contract was Mr. E. R. Givens, Technology Applications Division.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER USAAMRDL-TR-74-27	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD 782857
4. TITLE (and Subtitle) DESIGN AND DEVELOPMENT TESTING OF FREE PLANET TRANSMISSION CONCEPT		5. TYPE OF REPORT & PERIOD COVERED Final Report 26 Jun 72 - 31 Dec 73
		6. PERFORMING ORG. REPORT NUMBER CW-WR-73-040F
7. AUTHOR(s) Neil A. De Bruyne		8. CONTRACT OR GRANT NUMBER(s) DAAJ02-72-C-0113
9. PERFORMING ORGANIZATION NAME AND ADDRESS Curtiss-Wright Corporation One Passaic Street Wood-Ridge, New Jersey 07075		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project 1F162205A119
11. CONTROLLING OFFICE NAME AND ADDRESS Eustis Directorate U.S. Army Air Mobility R&D Laboratory Fort Eustis, Virginia 23604		12. REPORT DATE April 1974
		13. NUMBER OF PAGES 96
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Transmissions (Mechanics)
Gears

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents the results of an experimental program to demonstrate and evaluate the Curtiss-Wright free planet concept for power transmission.

The program consisted of designing a 500-horsepower speed reducer to operate at 8000 rpm input speed and 19.2425 reduction ratio. This design was procured and evaluated. The evaluation consisted of static and dynamic evaluation as well as 50 hours of endurance testing at rated operating conditions. Test

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20

evaluation was accomplished using a regenerative or back-to-back test arrangement. Results of the program are as follows:

A design of a 500-horsepower reduction gearbox with approximately 20:1 ratio was formulated for demonstration of the Curtiss-Wright free planet concept.

The static test consisted of obtaining gear tooth meshing patterns and planet spindle load distributions. The patterns showed good full face tooth meshing characteristics and verified the self-aligning hypothesis of the free planet concept. The planet spindle load distribution investigation showed excellent load distribution between the planet spindles.

Speed and load runs were conducted to show the dynamic capability of the free planet concept. These demonstrations were all successfully completed. At rated conditions efficiencies approaching 98% were achieved. At the end of the endurance program the efficiency slightly exceeded 98%.

Fifty hours of endurance testing were successfully completed at noted conditions of speed and load.

PREFACE

This final report covers the feasibility demonstration of the Curtiss-Wright free planet power transmission concept. This work included the design, fabrication and development demonstration of this type of advanced concept.

The program was conducted during the 18-month period from 26 June 1972 to 31 December 1973, for the Eustis Directorate, U.S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia, under Contract DAAJ02-72-C 0113, DA Project 1F162205A119, with joint funding by the Naval Air Propulsion Test Center, Trenton, N. J.

Technical direction was provided by Mr. R. Givens of the Eustis Directorate, U.S. Army Air Mobility Research and Development Laboratory, and by Mr. J. D. Conboy of the Naval Air Propulsion Test Center.

The program was conducted at the Wood-Ridge facility of the Curtiss-Wright Corporation by the Power Systems Department.

TABLE OF CONTENTS

	Page
PREFACE	iii
LIST OF ILLUSTRATIONS	vi
LIST OF TABLES	viii
INTRODUCTION	1
FREE PLANET CONCEPT	2
INTRODUCTION	2
GENERAL CONSIDERATIONS	2
DESCRIPTION	3
DESIGN AND DESCRIPTION OF DEMONSTRATION HARDWARE	16
TEST METHOD	29
STATIC	29
DYNAMIC	29
TEST RESULTS	35
STATIC	35
SPEED AND LOAD	40
ENDURANCE	40
ALTERNATE LUBRICATION SYSTEM	40
OVERSPEED AND OVERLOAD	43
EVALUATION AND ANALYSIS OF DESIGN AND TESTS	45
WEIGHT COMPARISONS	45
EFFICIENCY	46
CONCLUSIONS	48
RECOMMENDATIONS	49
LITERATURE CITED	50
APPENDIX. Design of Free Planet Demonstration Hardware	51
LIST OF SYMBOLS	79

LIST OF ILLUSTRATIONS

Figure		Page
1	Free Planet Schematic - Basic With Planet Forces	4
2	Power Hinge - Planet Forces to Three Planes	5
3	Free Planet - Tangential Plane Moment Balance for Facewise Load Distribution	9
4	Free Planet Schematics - Two-Stage Arrangements	11
5	Free Planet Schematic - Four Load Point With Planet Forces	13
6	Free Planet Schematic - Dual Rotation Outputs With Planet Forces	14
7	Free Planet Schematic	17
8	Free Planet Transmission FP500	19
9	Free Planet Transmission FP501	21
10	Free Planet Assembly	23
11	Spindle Assembly	24
12	Fixed and Output Ring Gears	25
13	Input Shaft Housing and Sun Gear	26
14	Test Rig With Fixed and Output Ring Gears	27
15	Test Housing	28
16	Strain Gauged Shaft and Readout	30
17	Control Panel	32
18	Setup for Recording Acoustic Emissions	33
19	Test Rig on Test Stand	34
20	Static Evaluation - FP500 Gear Tooth Load Patterns	38
21	Static Evaluation - FP501 Gear Tooth Load Patterns	39
22	FP500 Efficiency	41

Figure		Page
23	FP501 Efficiency	42
24	Free Planet Assembly After 50-Hour Endurance Test	44
25	Helicopter Main Transmission Weight Vs Torque-Rotor Output	47

LIST OF TABLES

Table		Page
I	Free Planet Nomenclature	7
II	FP500 and FP501 Gear Data	18
III	Planet Spindle Torque Variations FP500	36
IV	Planet Spindle Torque Variations FP501	36
V	Planet Spindle Torque Variations FP500	37
VI	Planet Spindle Torque Variations FP501	37
VII	Quantitative Analysis Transmission Comparison	46

INTRODUCTION

The requirements for transmissions for advanced helicopter and V/STOL aircraft have become increasingly demanding as requirements for lighter weight and greater reliability are continuously accentuated. The work summarized in this report addresses these requirements. This work represents a new concept in power transmission speed reducers and speed increasers. The concept is a derivative of the Curtiss-Wright power hinge design.

Prior to the contractual effort, engineering studies were conducted by the Curtiss-Wright Corporation to determine the feasibility of such a concept from a design point of view and to evaluate and compare this new transmission with currently available transmission systems. These studies have shown that, for example, in applications where multistage conventional planetary designs are normally employed, the new free planet concept offers the following potential advantages and benefits:

1. Improved reliability to the extent that it eliminates conventional planet bearings.
2. Lower weight
3. Reduced sensitivity to lubrication variations because of elimination of bearings and potential of greater survivability after loss of lubricant.
4. Fewer parts, less critical tolerances and lower overall cost.

The contractual effort reported herein is the second step of the development cycle, namely, designing, manufacturing, and demonstrating development units. The objective of this program was to verify some of the engineering work completed and to demonstrate the concept with real (load-carrying) hardware. These development units were rated at approximately 500-horsepower with approximately a 20:1 reduction ratio. The characteristics of this new drive system were investigated at loads and speeds compatible with current reduction gear applications.

FREE PLANET CONCEPT

INTRODUCTION

This discussion presents a new planetary gear speed changer concept; the principles involved, potential advantages, and preliminary designs of units for a variety of applications are presented.

GENERAL CONSIDERATIONS

The free planet transmission concept, as described in detail in the following section, covers a variety of planetary configurations which share the common characteristic that planet carriers or spiders are eliminated as are the conventional planet mounting bearings normally associated therewith. All forces and reactions are transmitted, whether through gear meshes or simple cylinders, in pure rolling contact.

In its simplest form this concept includes, as is shown schematically in the radial plane view of Figure 1, a fixed internal gear, X, a movable output internal gear, Z, and a number of planets. The concept illustrated in Figure 1 has C and B faces meshing with the corresponding internal gears, and an A face in mesh with an external sun gear. All planet faces are integral and are so spaced axially that gear tooth forces acting parallel to the tangential and transverse planes leave the planet in equilibrium when they are acting through the facewise centers of the various meshes. Tooth separating (and centrifugal) forces in the radial plane balance out between planets, all of which have diameters which roll on free, cylindrical rings, concentric with the sun gear axis. These rings are not shown in the schematic.

A fuller explanation of this simple form and an introduction to some of the alternative arrangements are included in the following section.

In applications where multistage conventional planetary designs would normally be employed, one or more of the following potential advantages may be found to make the concept attractive:

- Lower weight (typically 10-50% lighter) than conventional planetaries working at the same stress.
- Improved reliability to the extent that the bearings which it eliminates are sources of failure.

- Reduced sensitivity to lubrication variation because of elimination of bearings and potential of greater "survivability" after loss of lubricant.

- Suitability for use at very high speeds because of low "scoring indices" of internal meshes and ready means for lubricating/cooling even high-speed meshes. Speed not limited by bearing ratings of dynamics.

- Fewer parts and less critical gear tolerances with a possible reduction in costs. Flexibility can be built in, which minimizes the planet-to-planet load maldistribution from misindexing. Low scoring indices reduce the tooth surface finish requirements.

The following additional tentative generalizations are based on studies of several applications (helicopter transmissions, engine reduction gears, heavy winch drives, and miscellaneous rotary actuators):

- Efficiency is equivalent to competitive planetary systems. The increased gear losses of the differentially compound output gear meshes are offset by the elimination of support bearing losses.

- The overall envelope volume may be slightly larger than that of conventional planetary arrangements. However, the typically large hollow "core" provides volume which may be employed for lubricant storage/cooler or other purposes advantageous to the application.

- Not all configurations are adaptable to all applications; e.g., some require reaction anchorages which may be awkward to provide.

DESCRIPTION

The free planet transmission concept covers broadly those planetary gear arrangements wherein the planets are not constrained by being secured to a "spider" or "carrier". The Curtiss-Wright Power Hinge® is of this genre. See Figure 2. Here, radial forces (tooth separating components from gear engagement) are balanced out between the various pinions by free floating cylindrical support rings on which selected planet diameters roll. Planet skewing is prevented by matching, interconnected ring gears engaging the end (E) planet faces. These are opposed by symmetrically applied forces from the center (c) ring gear and the sun gear. Thus the planets are "free" in that they are constrained only by the gear meshes and the free-floating support rings.

Extension and elaboration of this basic philosophy have evolved a variety of novel configurations which have additional features of potential interest in a variety of "high ratio" planetary gear arrangements.

As a condition of equilibrium, forces and moments about any point in or parallel to three planes must add up to zero. As an example, the forces acting on a planet of a typical Power Hinge are illustrated in Figure 2.

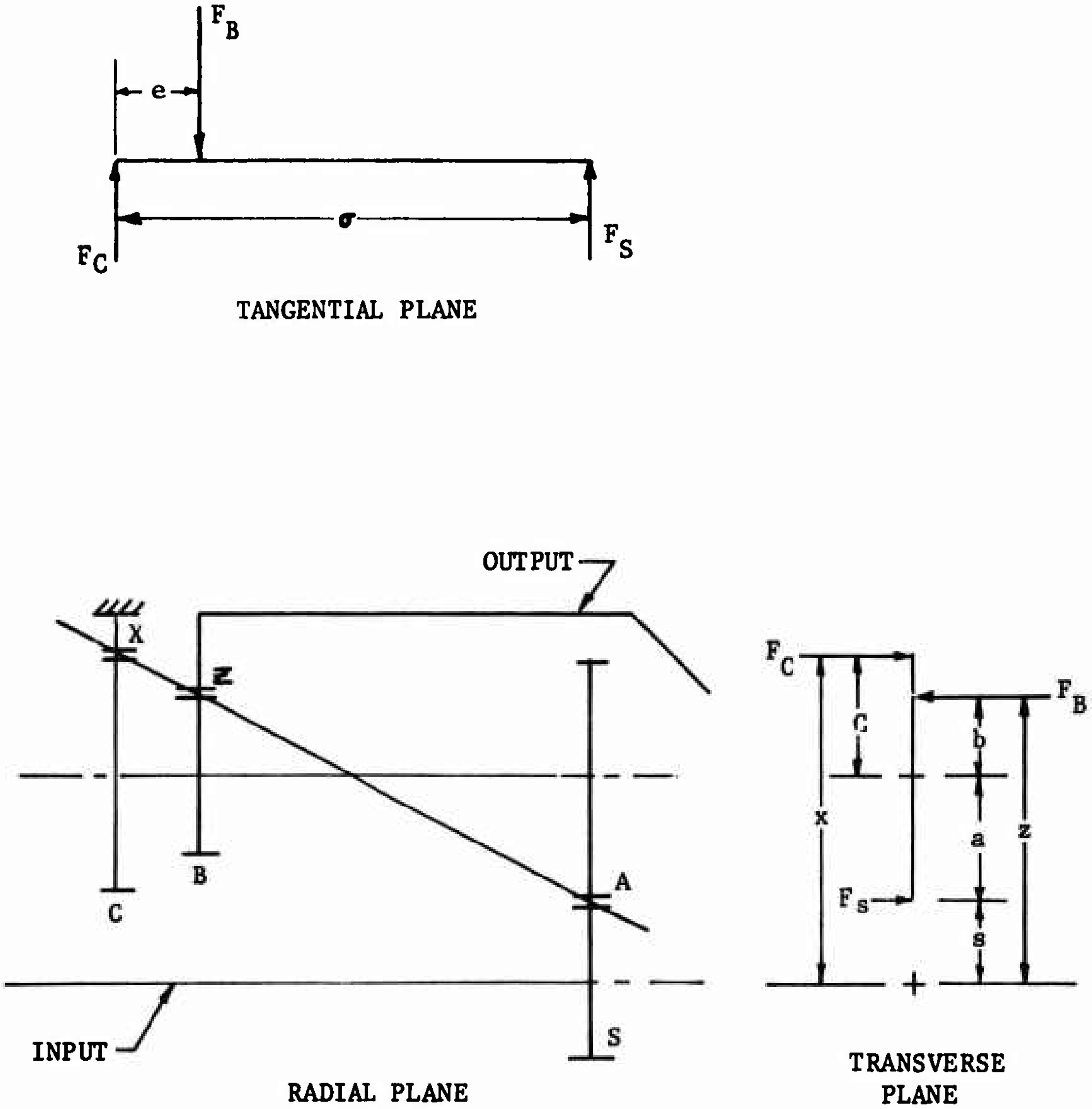


Figure 1. Free Planet Schematic - Basic With Planet Forces.

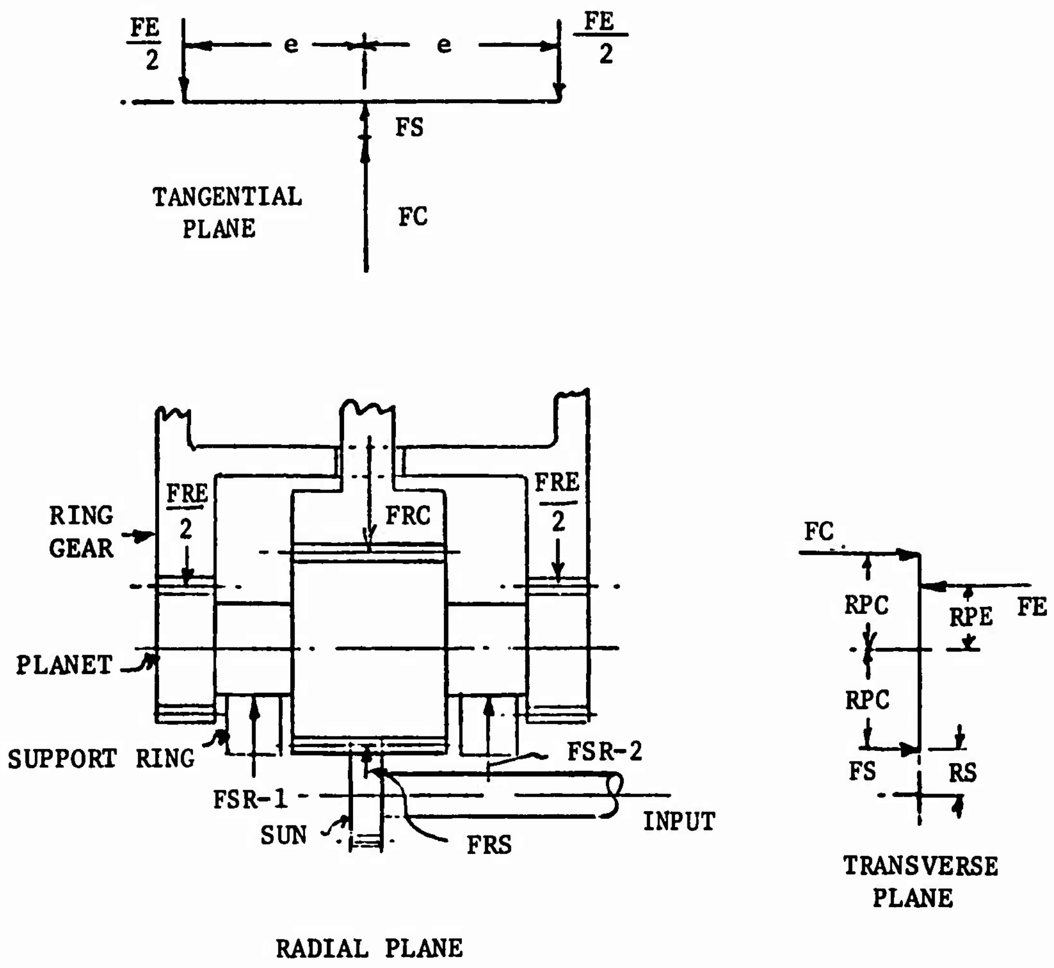


Figure 2. Power Hinge - Planet Forces to Three Planes.

In the radial plane, the net of the tooth separating forces (neglecting centrifugal forces, if any) is balanced by the reaction forces from the support rings, i.e., FRE + FRC - FS = FS R_1 + FS R_2. Assuming that the loads are applied at the centers of the tooth faces and rings, it is apparent that the moments about any point will also add up to zero in view of the symmetrical example employed.

Forces acting parallel to the transverse plane are defined by the load and the relative radii of the gears which have been selected to produce the desired reduction ratio as required by the application and FS + FC = FE, FS (RPC + RPE) = FC (RPC - RPE), FS (2 RPC) + FE (RPC - RPE), etc., for equilibrium.

It is clear that forces and moments acting parallel to the tangential plane are also balanced in this symmetrical arrangement.

The conditions of equilibrium required, fairly easily followed in this symmetrical example, can be similarly applied to unsymmetrical arrangements with more or fewer points of load application using the following rationale:

> The application defines the forces and the associated geometry in the transverse plane.
>
> Appropriate free-floating support (or retaining) rings react the loads in the radial plane.
>
> The axial distances between gear face centers must be established for equilibrium in the skewing direction from moments in the tangential plane.

While several additional fallouts have evolved, the above defines the bases which are the crux of the free planet concept.

The simplest nonsymmetrical example of the application of these principles is that illustrated schematically in Figure 2. (Nomenclature used in this example and in others to follow is outlined in Table I.) Support rings are ignored in these simplified sketches and hence also the radial plane forces which these balance out between planets.

From the transverse plane (Figure 2), we can define:

FS + FC + FB - Force Equilibrium

FB (C-b) = FS (C + a) - Moment Equilibrium About "C"

FC (C-b) = FS (a + b) - Moment Equilibrium About "B"

These equations establish the relative magnitudes of all tangential forces. The values at a, b, C, and s are appropriately selected based on geometric limitations and desired reduction ratio; for instance, assuming input at the sun gear, output from the Z ring gear with the X ring gear fixed,

TABLE I. FREE PLANET NOMENCLATURE

Quantity	Element	Symbols
Gears	Planets	A, B, C, G
	Sun	S
	Internal	X, Y, Z
Pitch Radius	"A" Planet	a
	"B" Planet	b
	"C" Planet	c
	"G" Planet	g
	"S" Sun Gear	s
	"X" Internal Gear	x
	"Y" Internal Gear	y
	"Z" Internal Gear	z
Axial Face	Distance Planet to Planet	e, σ , d
Diameter of Input Stage Drive Cylinder		d (2)
Diameter of Output Stage Drive Cylinder		D (2)
Torque		Q - (3)
Force	Tangential	F - (3)
Force (1)	Radial	FR -
Force (1)	Radial on Support Ring	FSR -
Ratio	Reduction Ratio	R
Pitch Radius (1)		R - (3)
Length	Planet Face Width	L - (3)
	Planet Face Width - Center	LPC
	Planet Face Width - End	LPE

(1) Used in Figure 1 only

(2) Used in discussion of Figure 4 (c) and 4 (d)

(3) Followed by A, B, S, etc., to identify gear or mesh

$$R = \frac{2\ a\ (s + a + b)}{s\ (x-z)},\ \text{etc.}$$

The forces so established are applied to the tangential plane to define in terms of e, which is at least half the sum of the "C" and "B" planet faces. Thus,

$$FB\ e = FS\ \sigma \quad \text{or} \quad FC\ \sigma = FB\ (\sigma - e)$$

where

$$e \geqq \frac{LPC + LPB}{2}$$

In all arrangements where a planet is in equilibrium and is acted on by three tengential forces,one can draw a "balance line" through these three points as shown in the radial plane, i.e., the three points of tangential load application lie in a straight line (in all planes). It is frenquently expedient to employ this "construction line" as an alternative way of conveniently establishing e and σ. In subsequent "three point" configurations, this line will be shown and can be considered the equivalent of mathematically establishing e and σ from the tangential plane balance equations. Where more than three points of tangential load application are involved, this simplification cannot be used.

The inherent tendency of a "free planet" to adjust its axis to produce uniform facewise tooth loading is illustrated in Figure 3(a), (b), and (c). In (a) the assumed uniform planet tooth loading is identified by a series of short arrows, while the resultant sum of these forces is shown for each tooth by a long arrow at the center of each tooth. Since the axial tooth spacing is defined by e and σ so that the net of moments in the tangential plane is zero, the planet is in equilibrium. If the planet axis is skewed as shown in (b) or (c), the resulting "end loading" on the teeth changes e and σ so that an unbalanced moment exists which tends to restore the planet axis to the condition of (a) where tooth loads are evenly distributed.

Not only does this phenomenon eliminate the necessity of moment and load absorbing planet carrier bearings, but it suggests that customary gear length to diameter limitations may be challenged without consequent corner loading.

The above assumes a number of ideal, identical planets operating on ideal, perfectly concentric mating gears. The implied merits must be examined in any specific, real application, to quantitatively examine the influence of expected imperfections as well as that of any detail construction adopted to nullify the undesirable effects thereof. A detailed quantitative analysis is possible only in a well-defined design and is, hence, not appropriate in this general description. However, anticipating analyses and constructions to be presented later, it can be said that the merits postulated do appear to be practically achievable. Effectively uniform facewise load distribution can be maintained even with long gear faces manufactured to moderate tolerances.

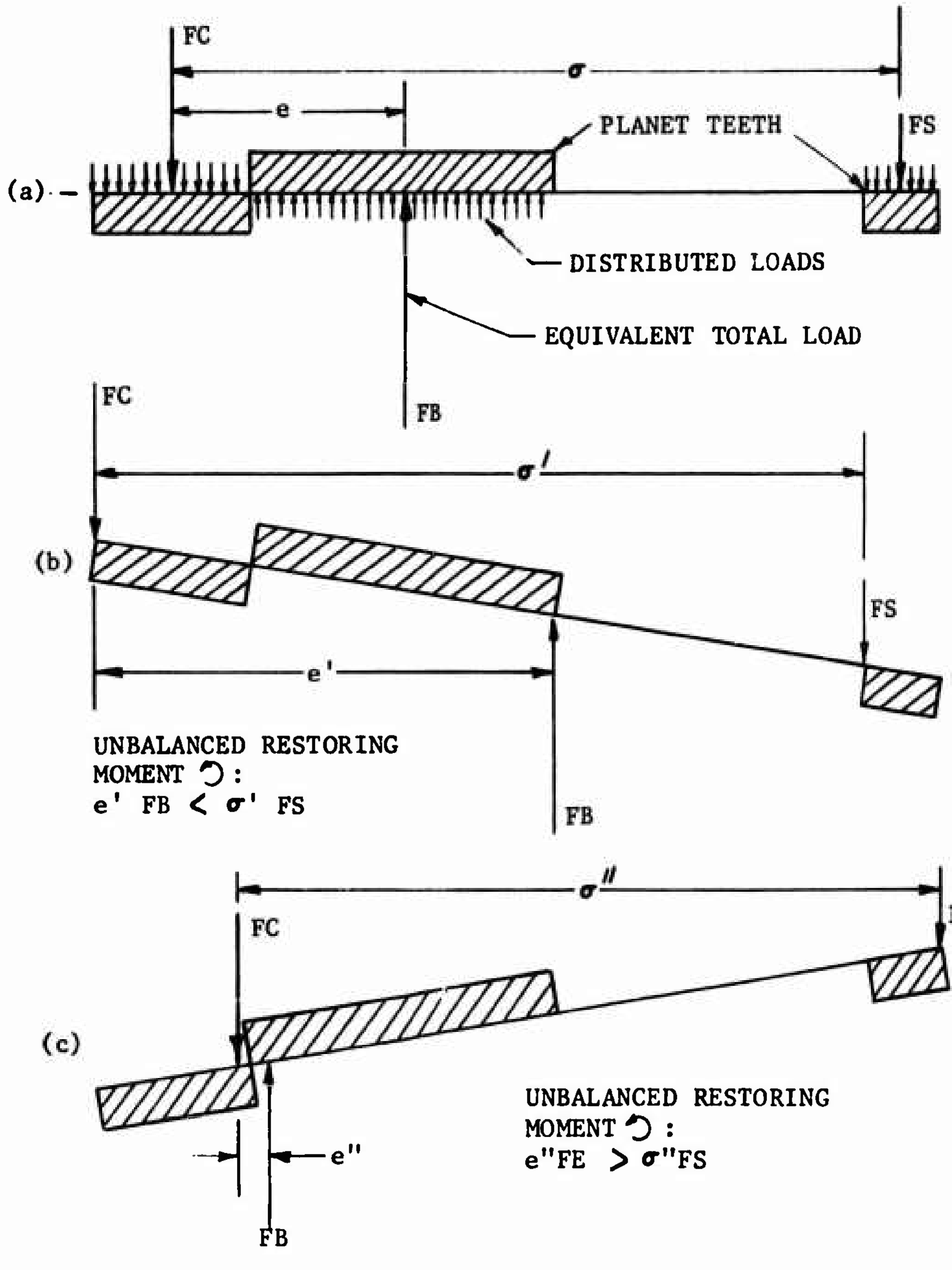


Figure 3. Free Planet - Tangential Plane Moment Balance For Facewise Load Distribution.

Figure 4 shows schematically (radial plane only) some two-stage variations in which each stage consists of a three-load-point planet. Figure 4(a) and (b), the simplest, show respectively "X" ring gear fixed and "Y" ring gear fixed equivalents of the corresponding version of the arrangement in Figure 1 previously treated. No additional reduction has been achieved but σ has been substantially reduced at the expense of providing a second fixed reaction ring gear. The second stage of the input is a simple spur gear journalled on an extension of the output stage planet at the intersection of its centerline and the balance line. If the pitch diameter of the second-stage input ring gear is the same as that of the output stage fixed ring gear, this bearing will have no gross rotation and will transmit only a tangential force (i.e, no torque).

Figure 4 (c) and (d) show arrangements which provide additional reduction in the input stage. The output stages are the same as that of Figure 4 (a) (the generally preferred arrangement). The additional reduction is provided by decreasing the pitch diameters of the fixed reaction ring and mating planet. This results in different rotational speeds of the input and output planets about their axis and requires that a bearing, or its equivalent, be interposed at the point (or in the plane) where the balance and centerline of the input and output stages intersect. Since it is desired that only a tangential force be transmitted at this point, a conventional bearing can be replaced by a roller of diameter "d" on the input stage rolling on the inside of a cylinder of diameter, D, on the output stage. By proper selection of the ratios of these diameters as a function of the gear diameters, pure rolling is assured.

As will be noted, Figure 4 (c) and (d) differ in that in (d) the S and G meshes are both at one end of the output stage, while in (c) the S mesh has been moved to the opposite end with the G and A planets being connected by a shaft running through the output stage.

Up to this point, discussion has been limited to three-load-point planets and combinations thereof in two stages. However, the basic concept is not limited to such arrangements, and in some applications it may be desirable to consider other configurations. Figure 5 illustrates one possibility. In the radial plane view it will be seen that an additional gear face, G, is located a distance, d, from the C face for the general purpose of shortening the planets. The transverse plane equations are:

$FC + FG + FS = FB$	Force Balance
$(FC + FG)\ (c-b) = FS\ (a + b)$	Moments About "B"
$(FC + FG)\ (x-s) = FB\ (a + b)$	Moments About "S"

These define the necessary relationships of (FC + FG), FS and FB. For balance in the tangential plane, e, d, and σ and the load division between FC and FG must satisfy the moment equations in this plane.

$(FG\ d = FS\ (\sigma-d) + FB\ (d-e)$	Moments Around "C"
$(FC\ d + FS\ \sigma = FB\ e$	Moments Around "G"

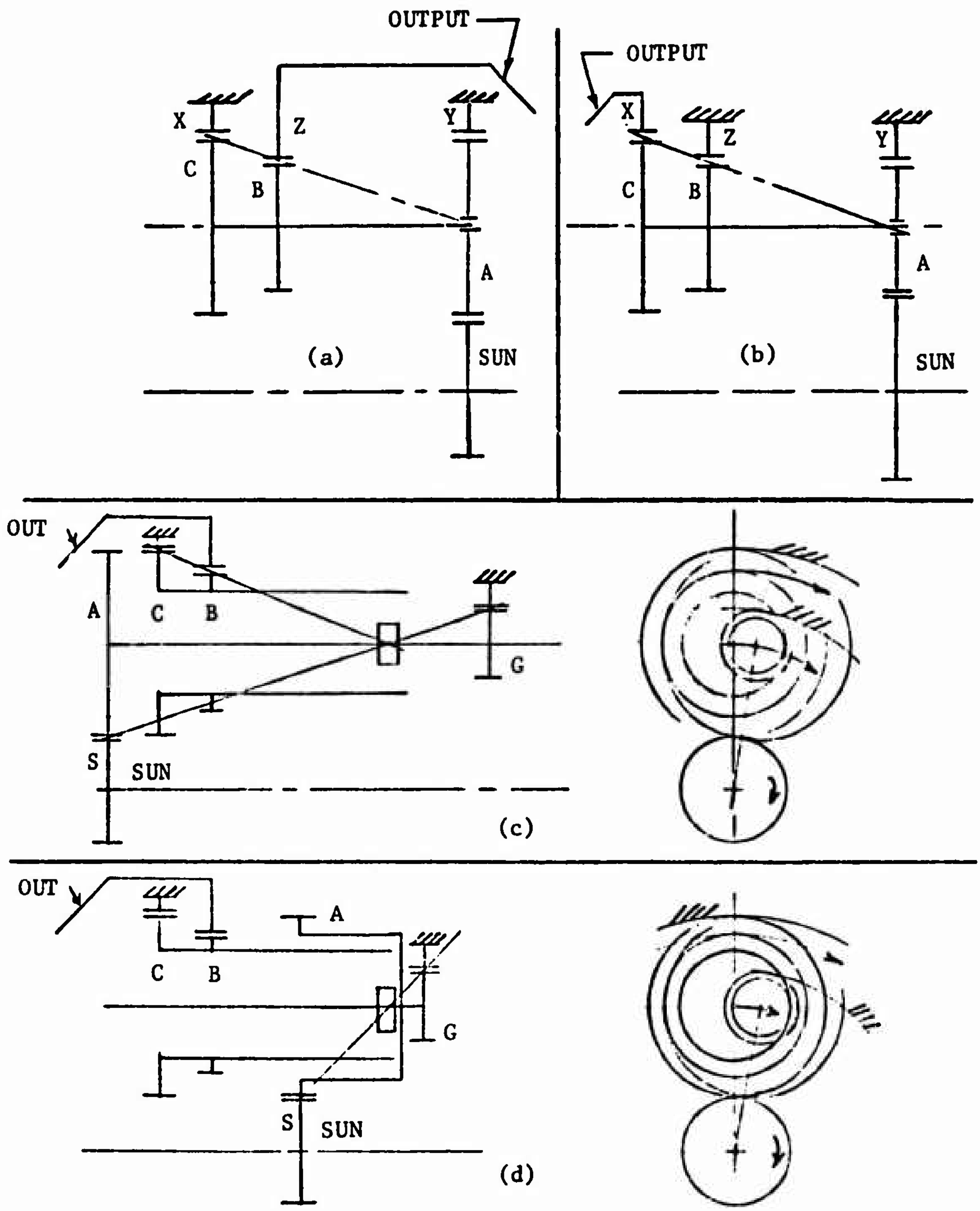


Figure 4. Free Planet Schematics - Two-Stage Arrangements.

These do not, however, define specific values, and one is free to choose e and d, solving for σ, at which time the proportion of the fixed reaction load shared by C and G will be established. It remains to provide means to insure this load division. Probably the simplest is to provide two of the meshes, say S and G, as shown in Figure 5 (b), with helical teeth of the appropriate helix angles and tends to load the G mesh to the level desired in accordance with the relationship, FS Tan α S = FG Tan α G. Balance is then maintained by slight axial shifting of the planet to produce the needed FG.

It is interesting to note that in this example of the four-load-point planet, no total additional gear face is required nor are additional gearing losses produced. The same total forces and moments are involved as in a comparable three load point system. The added G mesh merely takes over a portion of the C or B mesh load, but in a way to balance the planet in the tangential plane.

Other arrangements are possible. The G mesh can be placed on the other side of the C mesh (in effect making d negative) where the direction of FG would be reversed. Or other gear faces can be made helical to establish proper moment loading in the tangential plane.

Of the many possible combinations made available by the free planet concept, only a few have been outlined. However, there is one additional configuration which should be included: one that is evolved for dual rotation applications - where two oppositely rotating outputs are desired. One version is shown schematically in Figure 6. Here there is only one fixed reaction member, the X-ring gear, which takes out the torque introduced by the sun gear input, thus allowing the oppositely rotating outputs from the C and B ring gears to have precisely equal torques in the balanced condition. It is assumed that they also have equal speeds or reduction ratios, RY and RZ = R. A summary of the equations defining the controlling relationships follows:

From the transverse plane view:

$$FC\ Y = FB\ Z \qquad \text{Equal Output Torques}$$

$$RY = \frac{y\ (a + g)}{s\ (x - y)} \qquad \text{Ratio of Y Output}$$

$$RZ = \frac{z\ (a + g)}{s\ (x - z)} \qquad \text{Ratio of Z Output}$$

Since these are specified as equal, one can solve for:

$$g = \frac{zc + yb}{y + z}$$

$$FS\ (a + g) = FC\ (c - g) + FB\ (g - b) \qquad \text{Moments About "G"}$$

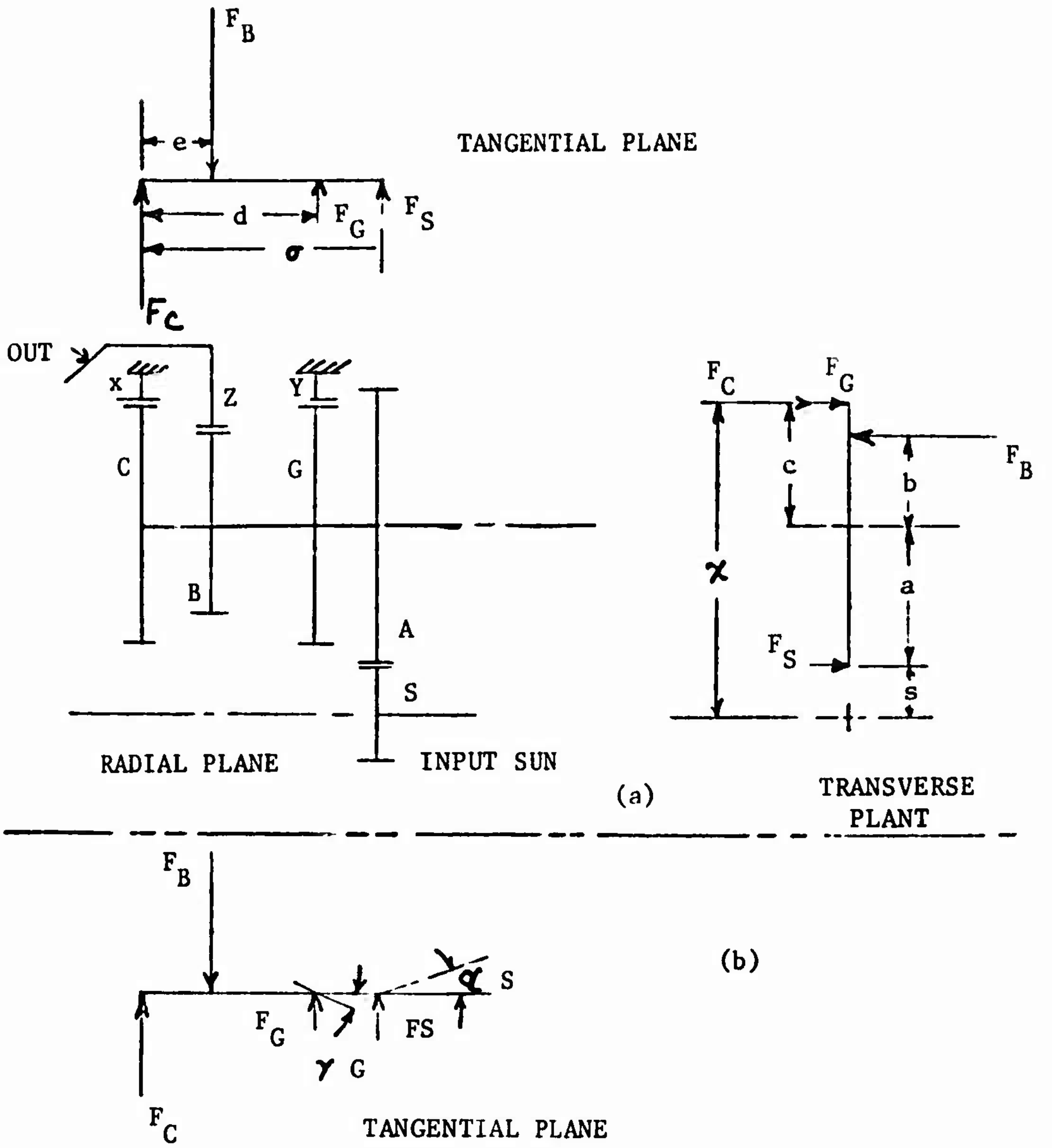


Figure 5. Free Planet Schematic - Four Load Point With Planet Forces.

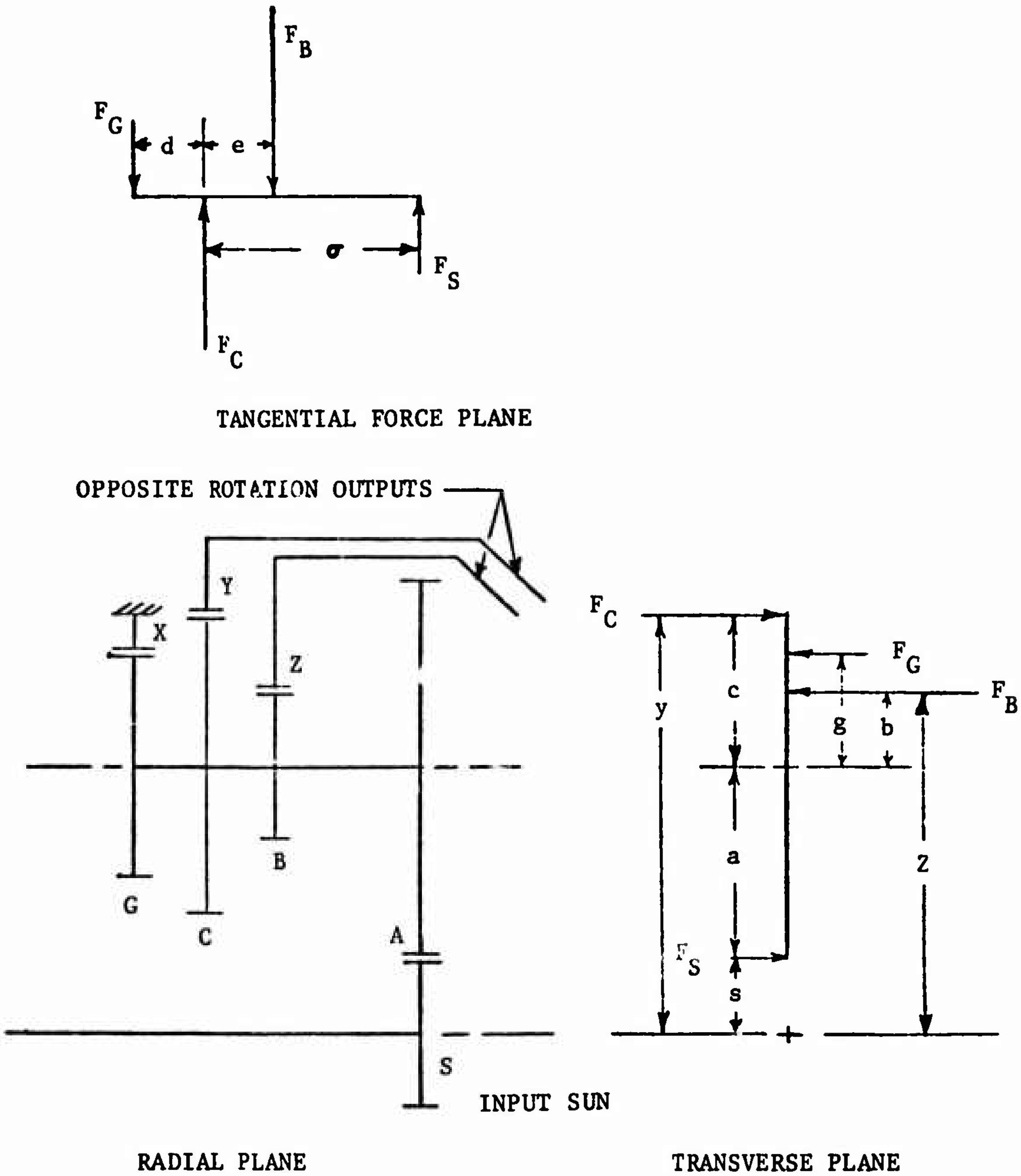


Figure 6. Free Planet Schematic - Dual Rotation Outputs With Planet Forces.

Substituting above derived values for g and FB gives:

$$FS = FC \frac{2y(c-b)}{y(a+b)+z(a+c)}$$

$$FG + FB = FC + FS \qquad \text{Summation of Forces}$$

Substituting the above values of FS and FB in the summation of forces, FG can be determined in terms of FC.

$$FG = FC \left(1 + \frac{2y(c-b)}{y(a+b)+z(a+c)} - \frac{y}{z}\right)$$

This completes definition of the force relationships established by the application. These can be viewed in the tangential plane to achieve balance by proper selection of e, σ and d in the equation:

$$FG\,d + FS\,\sigma = FB\,e \qquad \text{Moments About "C"}$$

Once again, this does not establish unique d, e and σ values but defines the necessary relationship of two when the third has been selected and the value of one when two have been picked. This will generally follow the pattern, select e based on (LPC + LPB)/2, place <u>d</u> conveniently from geometric considerations and LPG and finally solve for σ .

For this configuration there is no requirement for helical gears. Since FG is fixed when the output torque demands are, for any reason, not equal, uneven load distribution will result. If this unbalance is not severe or of long duration, a nominal increase in output gear tooth strength should compensate for this unbalance.

The arrangement shown above for the dual-rotation free planet application assumed equal output speeds and torques. The reaction torque of the G mesh balances the input torque at the S mesh. Should the occasion arise, variations of this scheme can be applied to unequal torques and/or speeds by appropriately changing <u>g</u> and <u>d</u>. If QB = QS + QC, the G mesh can be eliminated altogether or can be of a type intermediate between Figure 4 (a) and (b).

DESIGN AND DESCRIPTION OF DEMONSTRATION HARDWARE

The free planet concept has been demonstrated in this program utilizing two sets of hardware designated FP500 and FP501. The detailed design review for these configurations is presented in the Appendix. A summary of the designs and pertinent design data is presented in this section. The demonstration hardware provided a reduction ratio of 19.2425. The rating of this gearbox is 500-horsepower with an input speed of 8000 rpm.

The FP500, the basic test unit, is a planetary gear assembly which is compounded and consists of three tear meshes. The first plane of the planet gear meshes with the sun gear, the second plane meshes with the output internal ring gear, and the third plane meshes with a stationary internal ring gear. This configuration is shown schematically in Figure 7. The first and second planes of the planet gear are splined, double piloted, and locked to a quill shaft by a nut and cup lock. The third plane is splined, double piloted, and locked to the second plane of the planet gear by a nut and cup lock. The gears are timed so that the second and third plane planet gears have a tooth in line at the top vertical centerline and the first plane planet gear has a tooth in line 180° at the bottom vertical centerline.

The FP501 is essentially the same as the FP500 except that the quill shaft has been eliminated and torque is transmitted through the hollow support shaft.

Figure 8 shows the FP500 layout, and Figure 9 shows the revisions to make the FP501 layout drawing. Each of these configurations utilizes five planet spindles. Pertinent gear data is tabulated in Table II.

Pictures of the demonstration hardware are presented in Figures 10 through 15. Figure 10 shows the free planet assembly with the five planet spindles, roller support rings sun gear input, and the fixed and output ring gears. Figure 11 shows a planet spindle assembly and Figure 12 shows a planet spindle assembly and Figure 12 the fixed and output ring gears. Figure 13 is the input shaft, sun gear, and internal spindle support ring. Figures 14 and 15 show the test housing and torque drum.

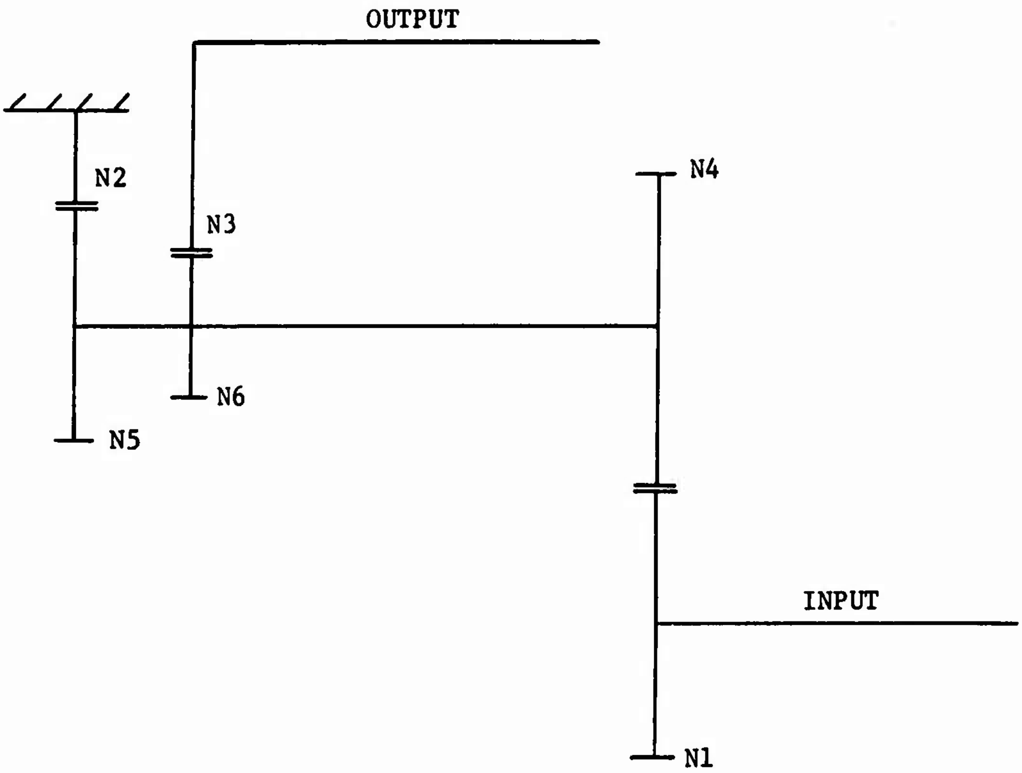


Figure 7. Free Planet Schematic.

TABLE II. FP500 AND FP501 GEAR DATA

	GEAR					
	N1	N2	N3	N4	N5	N6
NUMBER OF TEETH	66	158	143	79	43	28
PITCH DIAMETER, INCHES	5.2345	15.8	14.3	6.2655	4.3	2.8
PRESSURE ANGLE, DEGREES	20	20	20	20	20	20
DIAMETRAL PITCH	12.6087	10	10	12.6087	10	10
CONTACT RATIO	1.979	1.926	1.876	1.979	1.926	1.876
BENDING STRESS @ RATED CONDITION	14.7K	31KSI	26.5KSI	14.7K	31KSI	26.5KSI
CONTACT STRESS @ RATED CONDITION	83K	195KSI	135KSI	83K	135KSI	135KSI

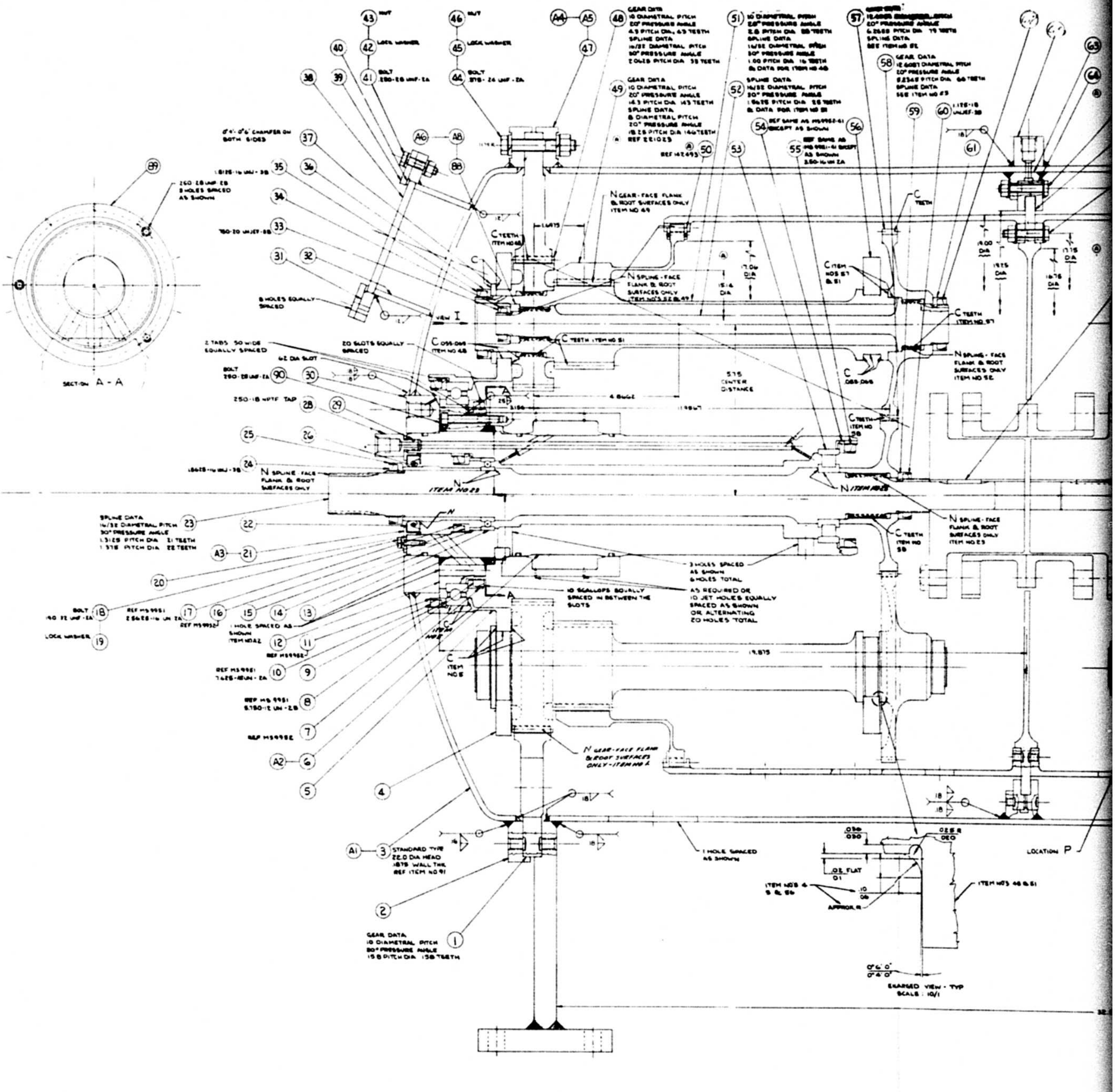
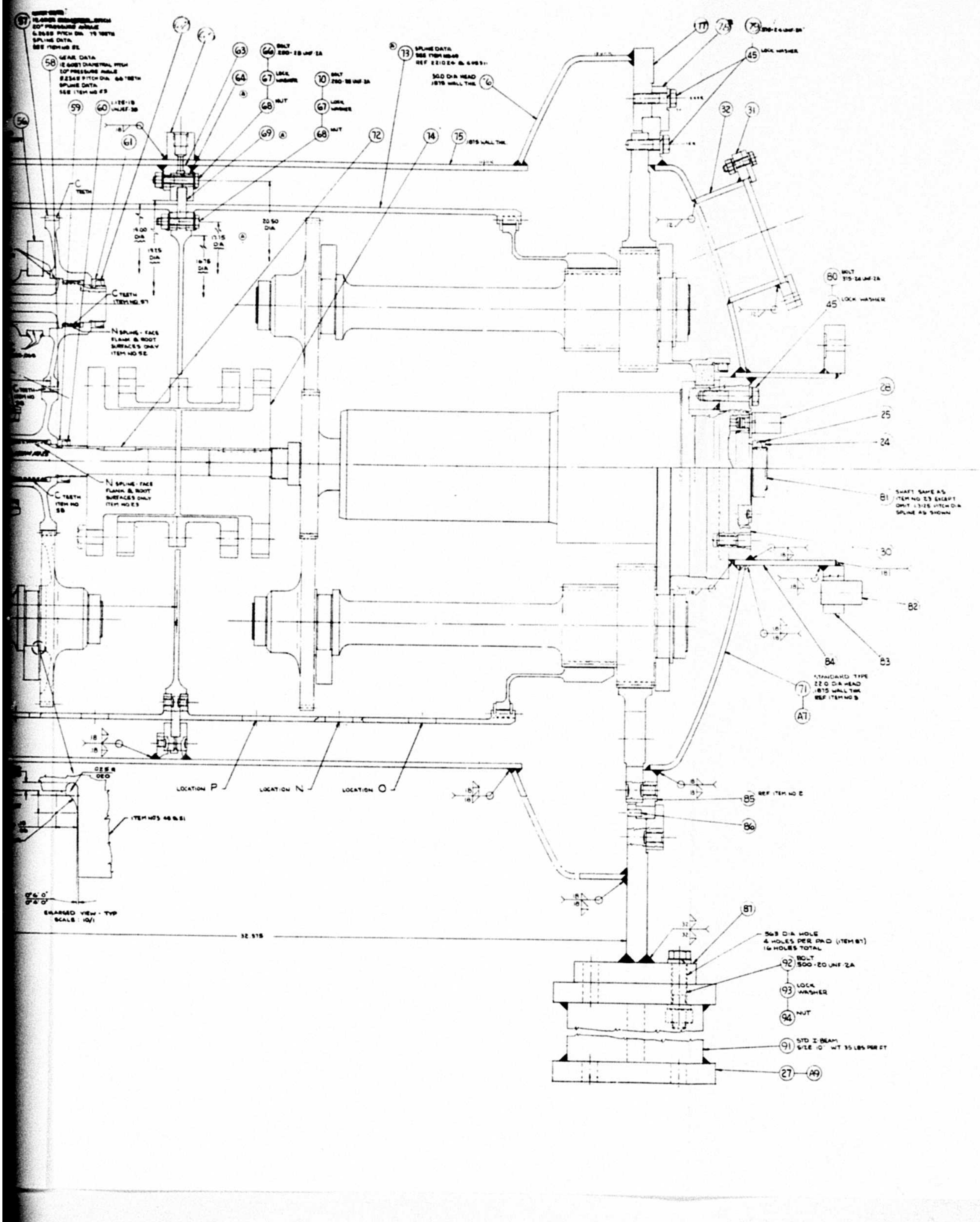


Figure 8. Free Planet Transmission FP500.



LOCATION P
LOCATION N
LOCATION O
LOCK WASHER
SHAFT SAME AS ITEM NO 23 EXCEPT
STANDARD TYPE
22.0 DIA HEAD
.1875 WALL THK
REF ITEM NO 8
563 DIA HOLE
4 HOLES PER PAD (ITEM 87)
16 HOLES TOTAL
BOLT
500-20 UNF-2A
LOCK WASHER
NUT
STD I BEAM
ENLARGED VIEW - TYP
SCALE 10/1
32.375

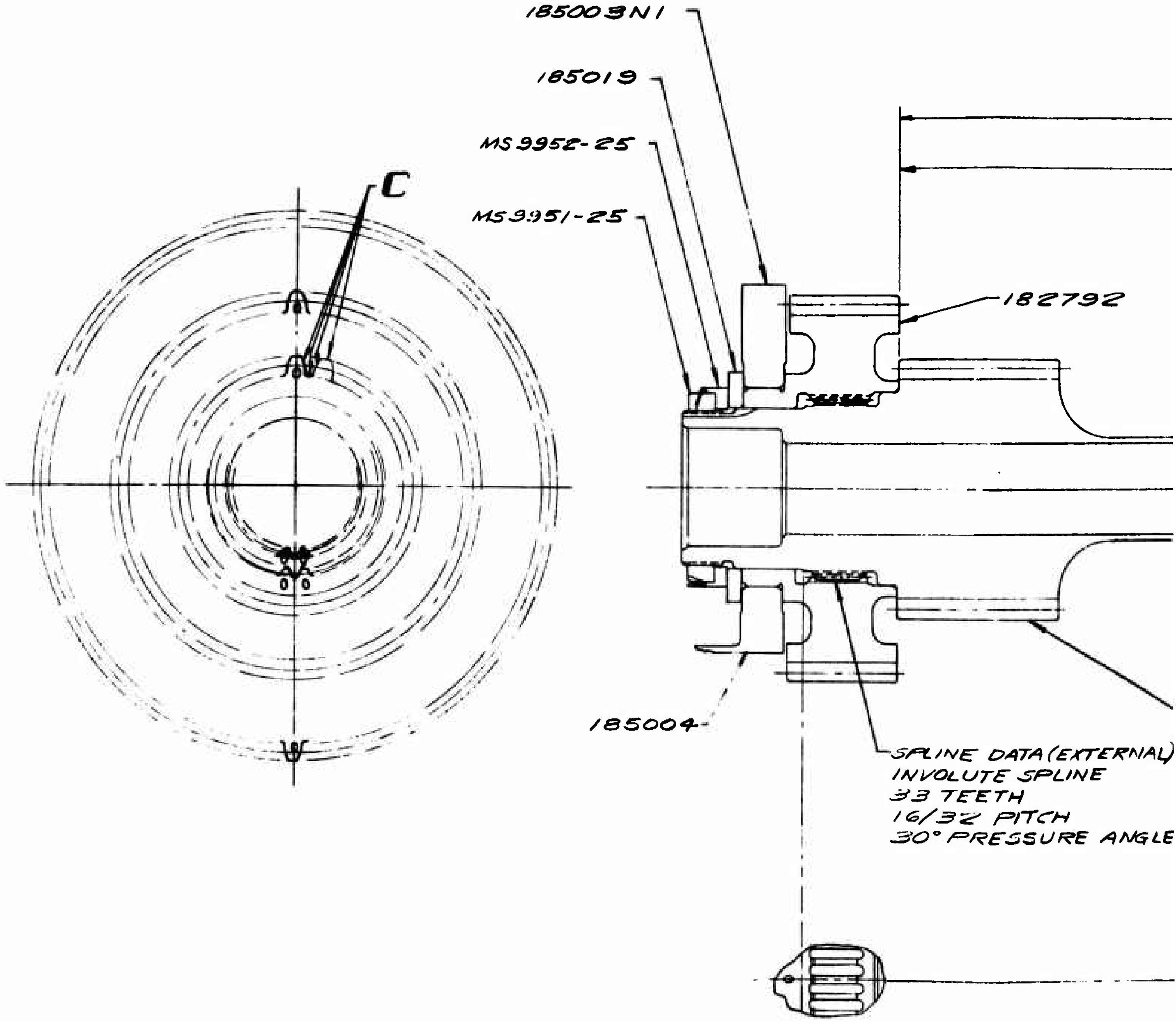
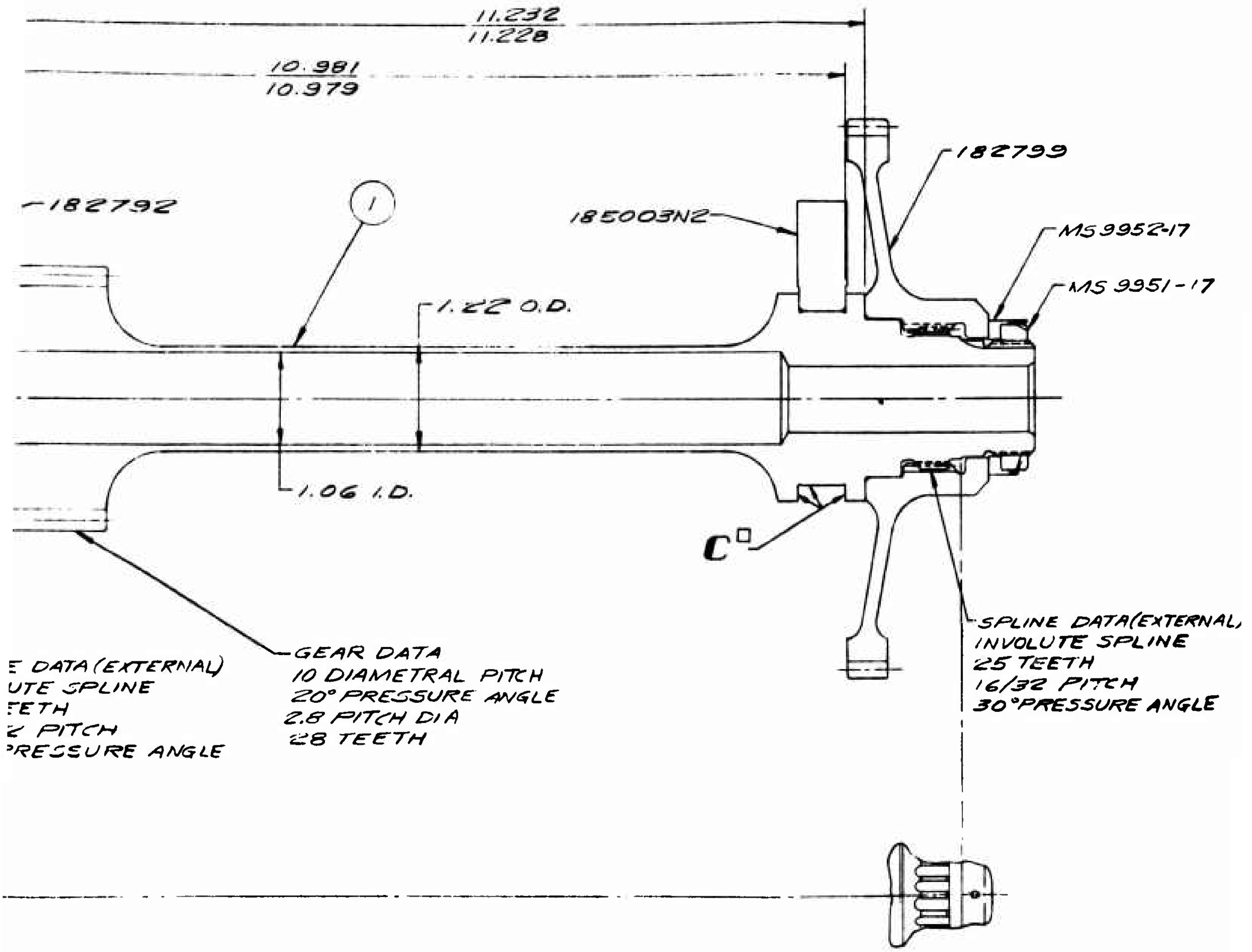


Figure 9. Free Planet Transmission FP 501.



11.232
11.228
10.981
10.979
182799
182792
185003N2
MS 9952-17
MS 9951-17
1.22 O.D.
1.06 I.D.
C
SPLINE DATA(EXTERNAL)
INVOLUTE SPLINE
25 TEETH
16/32 PITCH
30° PRESSURE ANGLE
GEAR DATA
10 DIAMETRAL PITCH
20° PRESSURE ANGLE
2.8 PITCH DIA
28 TEETH
E DATA (EXTERNAL)
UTE SPLINE
EETH
2 PITCH
PRESSURE ANGLE

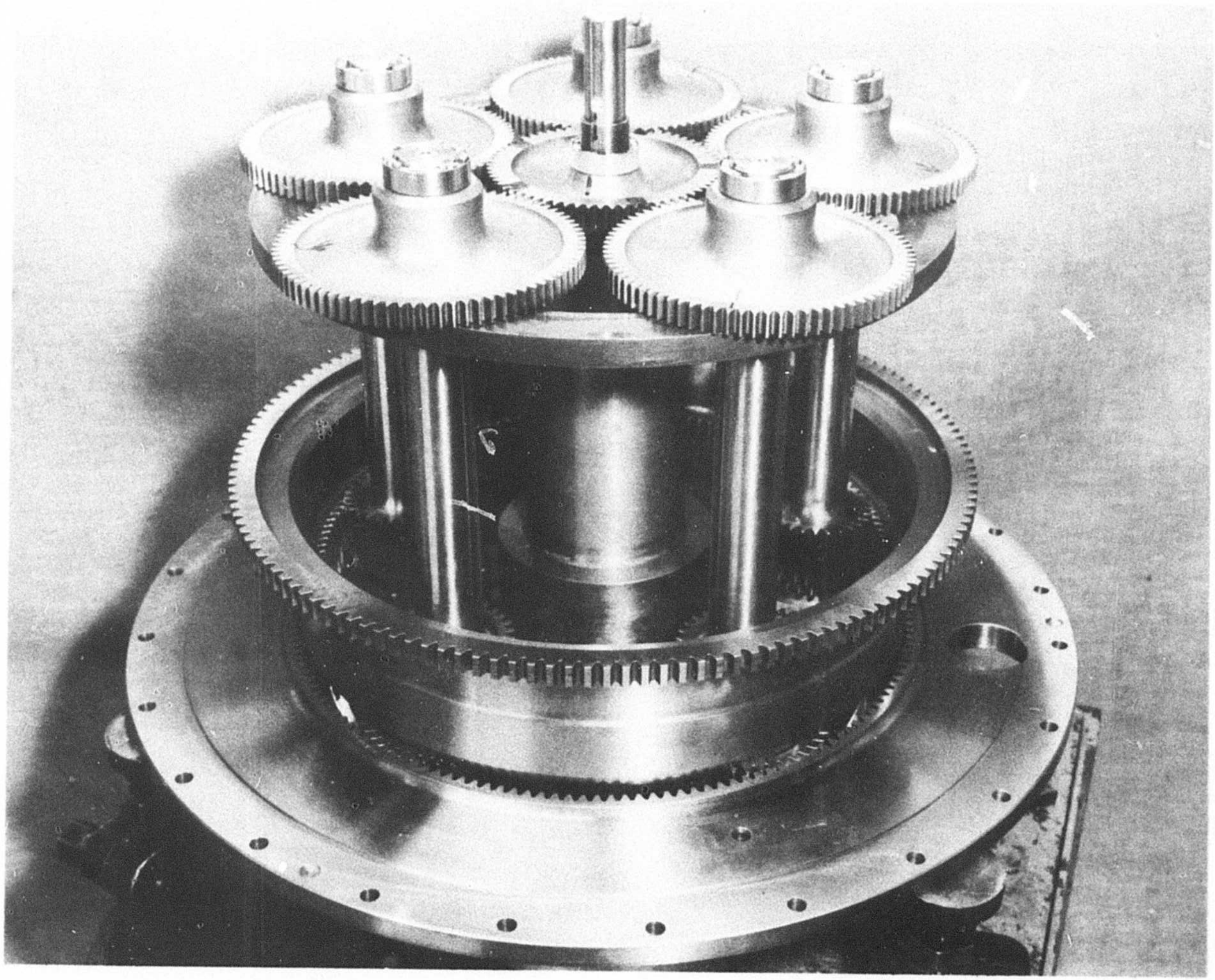


Figure 10. Free Planet Assembly.

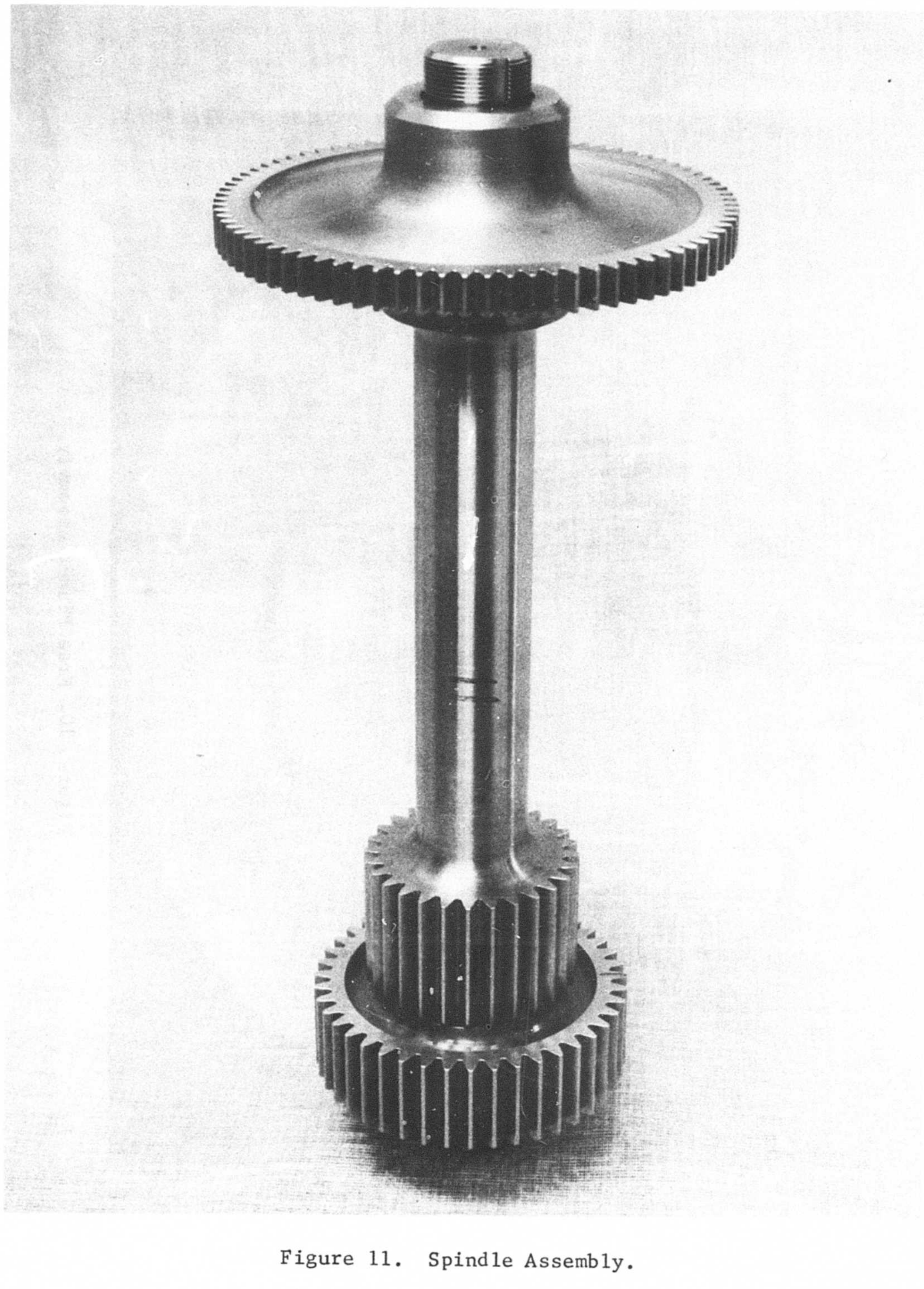


Figure 11. Spindle Assembly.

Figure 12. Fixed and Output Ring Gears.

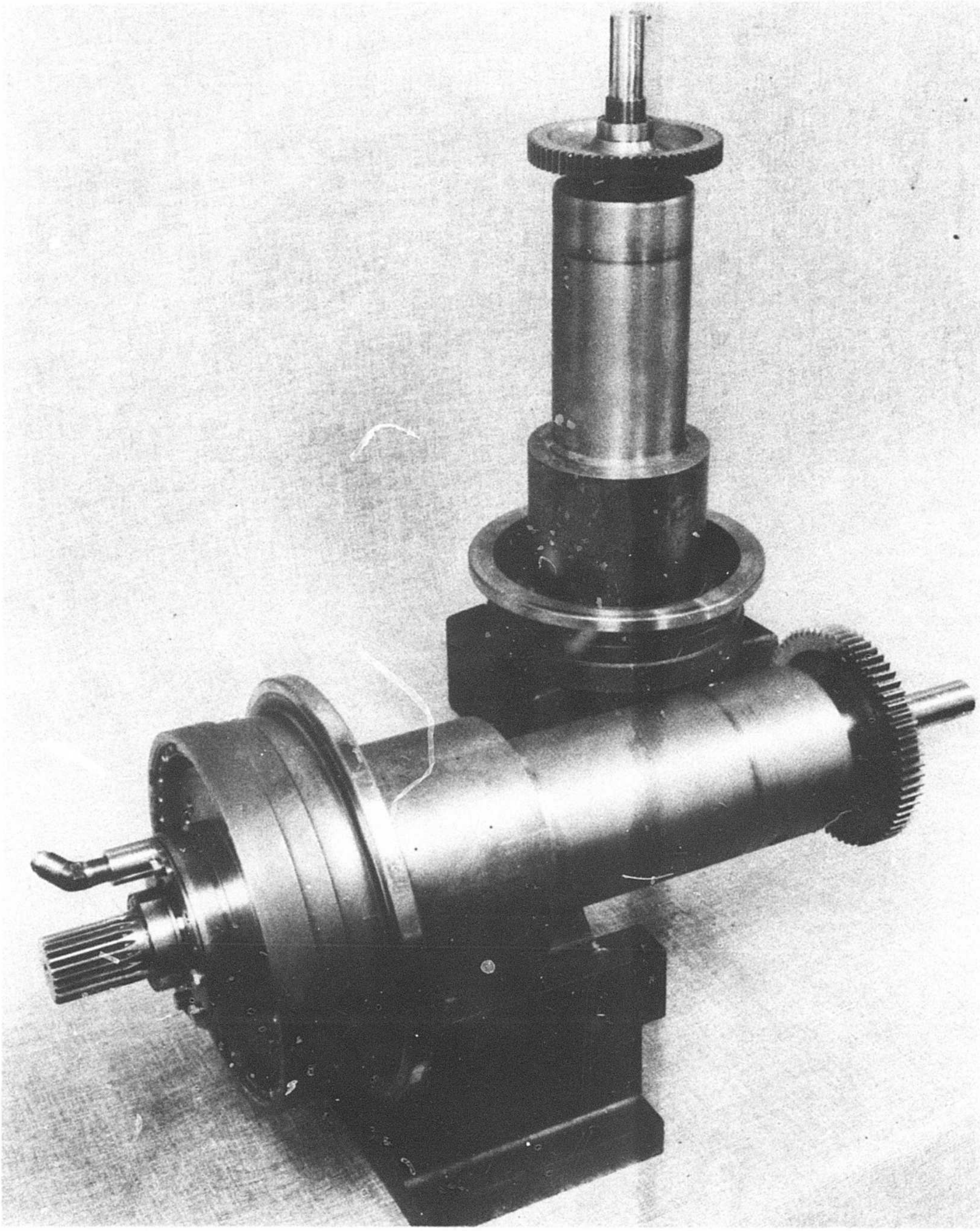


Figure 13. Input Shaft Housing and Sun Gear.

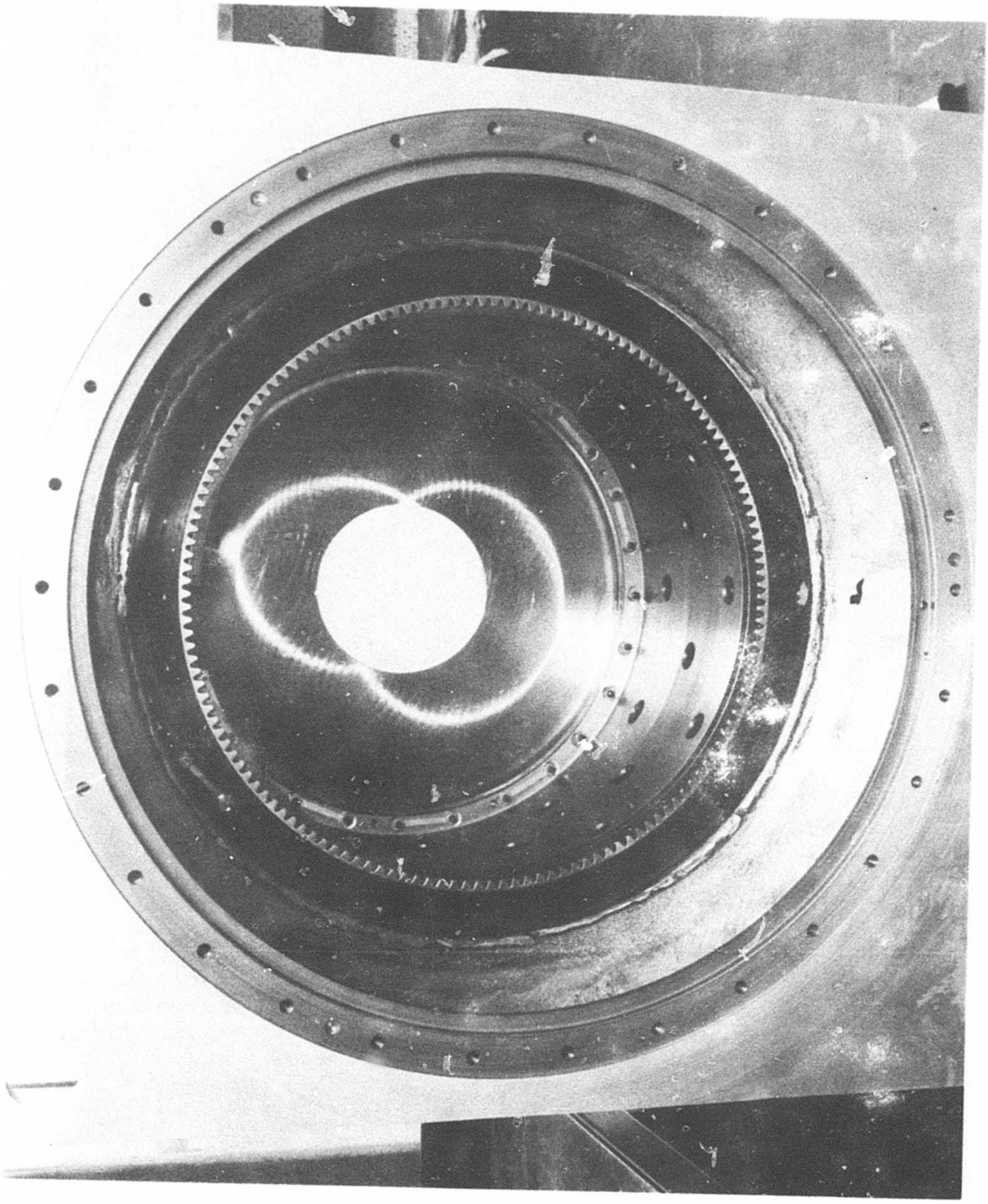


Figure 14. Test Rig With Fixed and Output Ring Gears.

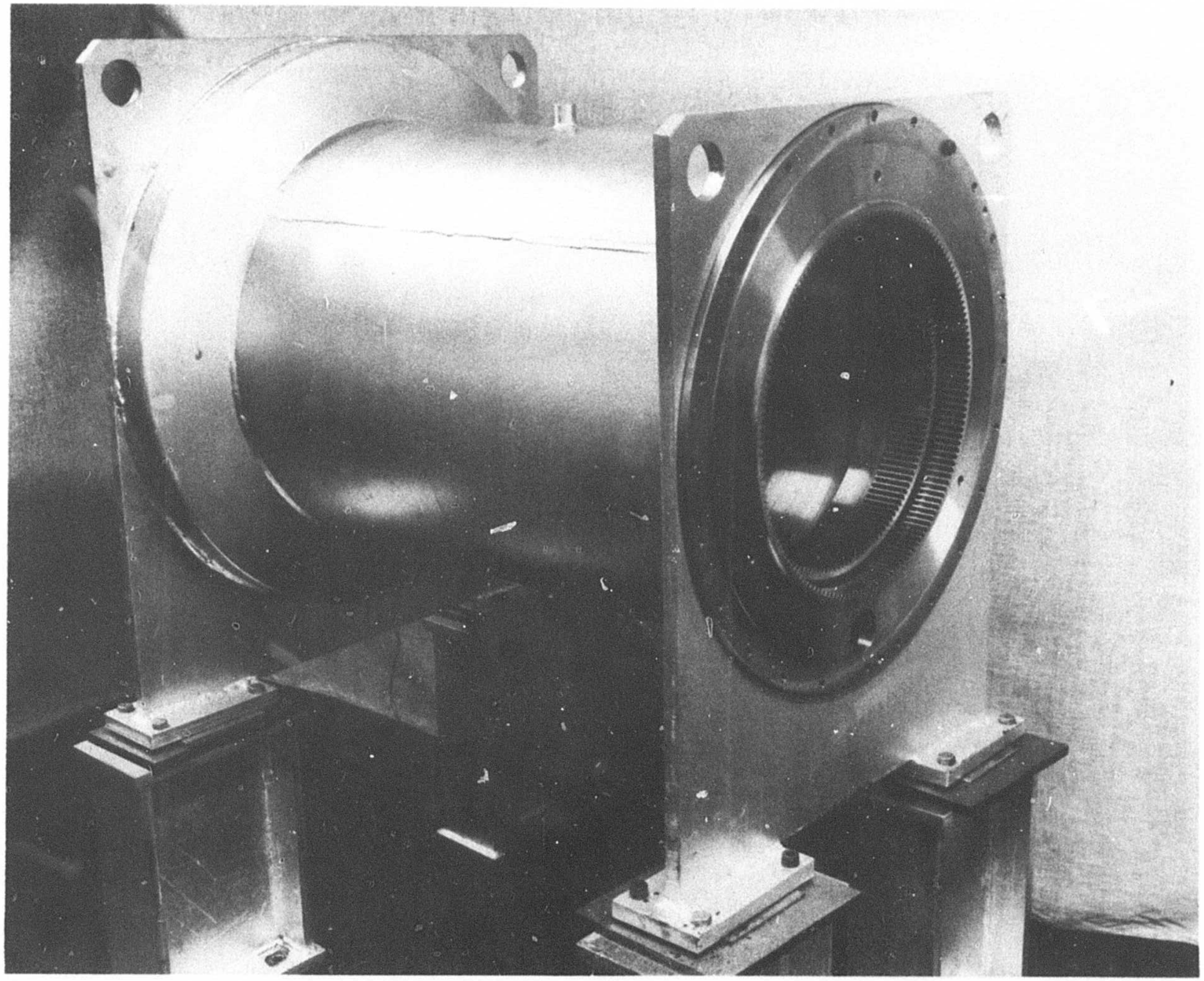


Figure 15. Test Housing.

TEST METHOD

STATIC

Static testing was conducted prior to dynamic testing to provide initial verification of the principles involved in operation of a free planet transmission. There were two separate evaluations conducted in this phase: They were strain gauge and tooth pattern evaluations.

In order to ascertain the load distribution among the planets, a static strain gauge test was performed. This test consisted of instrumenting the 5 planet quill shafts and instrumenting the output or rotating ring gear. An instrumented quill shaft and a strain gauge readout are shown in Figure 16. Prior to assembly into the test unit, each instrumented quill shaft was calibrated over the complete operating torque range. This provides a basis for determining the static torque each shaft is carrying in the assembled test unit. The unit was then assembled into the test rig, statically torqued over the complete operating range to 125% of rated torque. Data was recorded at 25, 50, 75, 100 and 125% of rated torque. In order to verify this critical measurement, the movable ring gear was also instrumented, and strain gauge readings were taken as the various planets were aligned with the ring gear strain gauges. These readings also indicated the equality level of torque as the various planet gears passed the ring gear strain gauges.

In the second phase of the static test program,gear tooth bearing patterns were established. The procedure for this consisted of checking the gear tooth load patterns at load increments of 25, 50, 75, 100 and 125% of rated torque. A red lead compound was applied to each gear set before each test, and the impression or pattern was read from the red lead. A sample of each mesh pattern was then "lifted" using transparent tape to provide a permanent record. The purpose of this test was to indicate if there was any cornering or abnormal load pattern on any gear elements and,more importantly, to verify the self-aligning hypothesis that the free planet is designed against.

DYNAMIC

A back-to-back or regenerative arrangement was used for the test evaluation of the FP500 and FP501 transmission. In this configuration the input or sun gear shafts of the two transmissions are fixed to each other by a coupling,with one shaft extending beyond the rig housing to a drive shaft. The output ring gears are also connected to each other by a splined drum. The direct connecting of the ring gears is a simplified form of attaching output shafts to the output ring gears and coupling the shafts. The nonrotating or fixed ring gears attached to the transmission housing absorb the reaction torque. One of the fixed ring gears was rotated by a torque arm

Figure 16. Strain Gauged Shaft and Readout.

to introduce and lock torque into the planetary gearing system. The hydraulic actuator loading the torque arm was calibrated in order to indicate the torque in the planetary gearing.

The test stand control panel is shown in Figure 17. During all testing, constant recordings of the acoustic emission were made by the equipment shown in Figure 18. These recordings were made to aid in diagnosing any problems that could occur during testing. A second view of the test rig mounted on the test stand, showing the torque arm and loading ram, is shown in Figure 19.

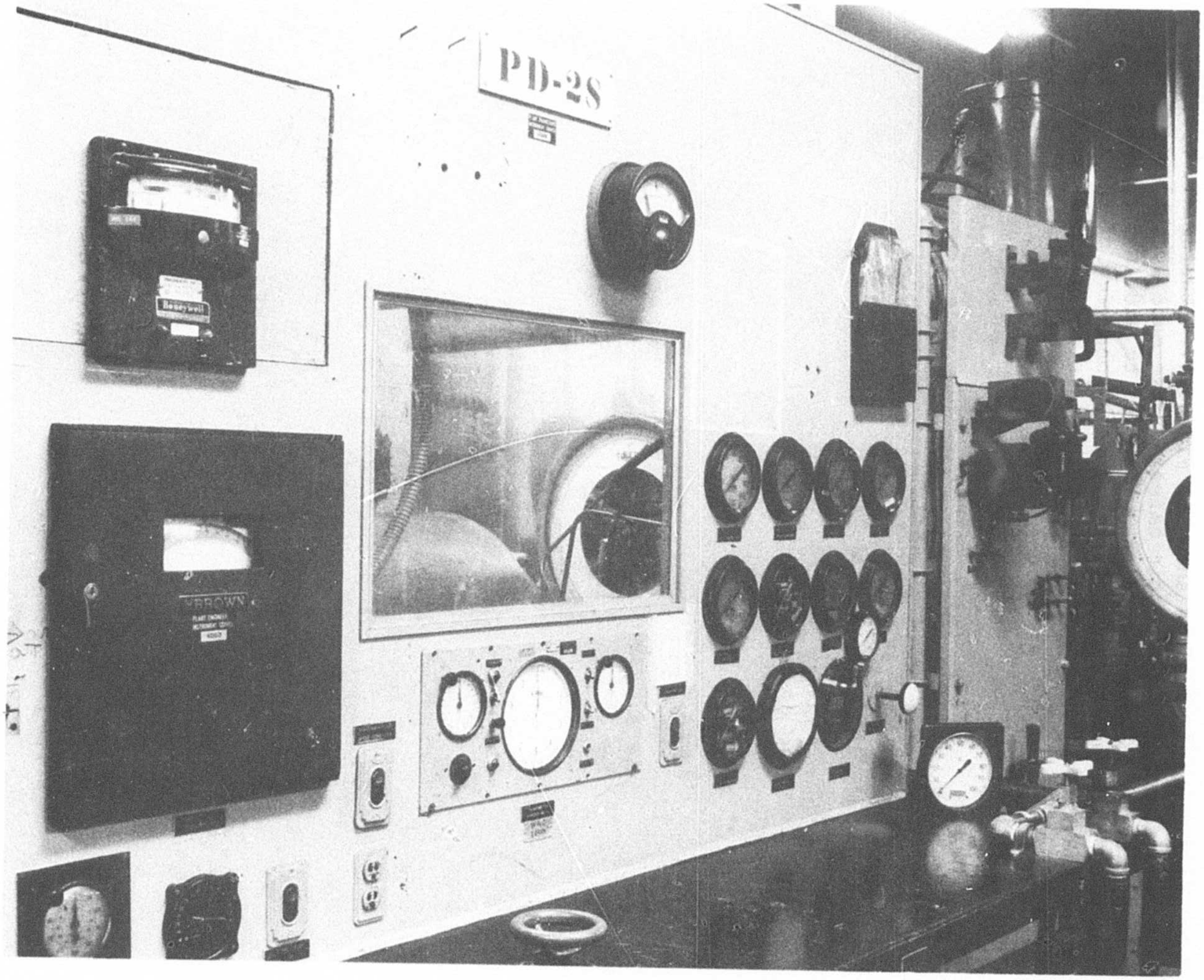


Figure 17. Control Panel.

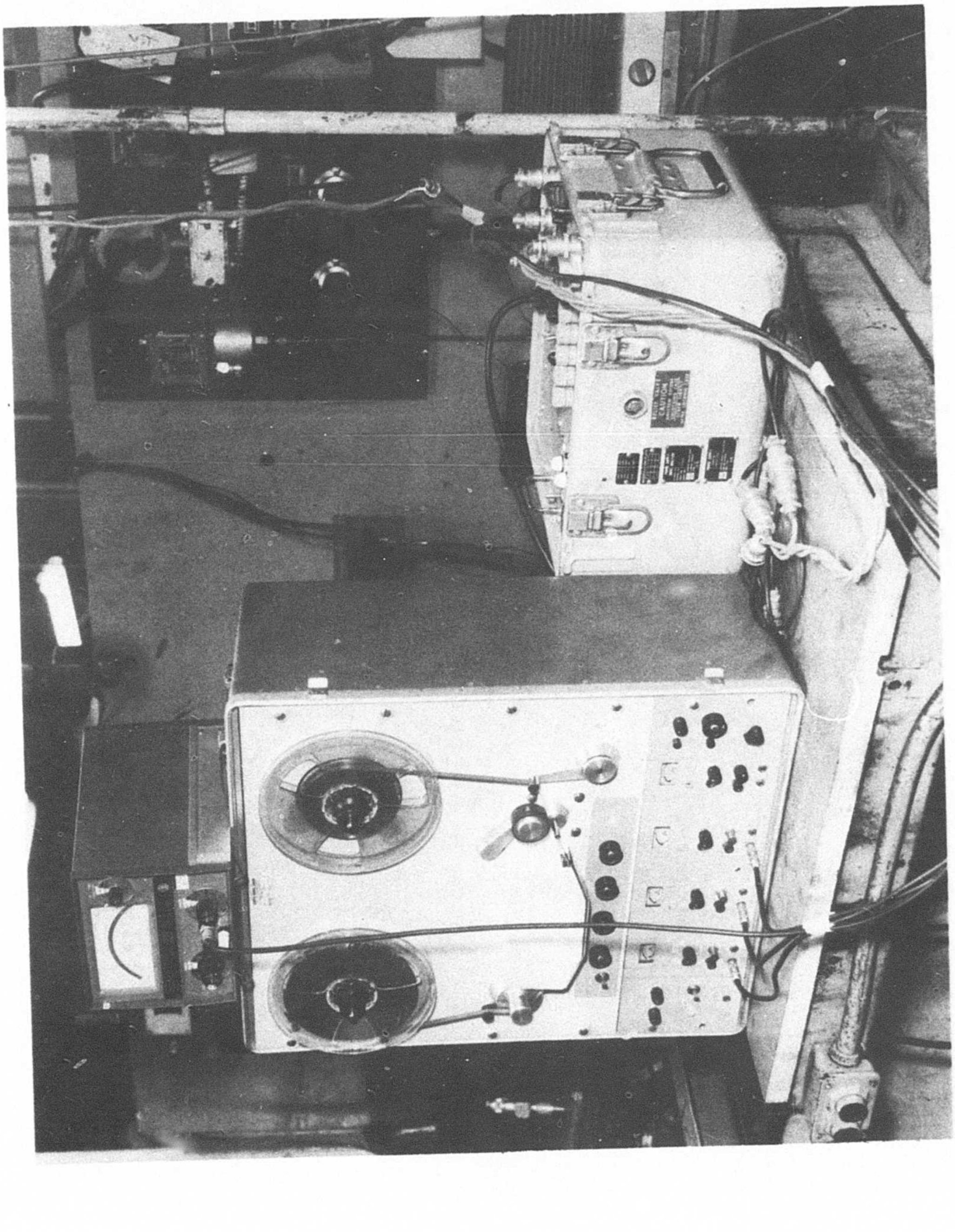


Figure 18. Setup for Recording Acoustic Emissions.

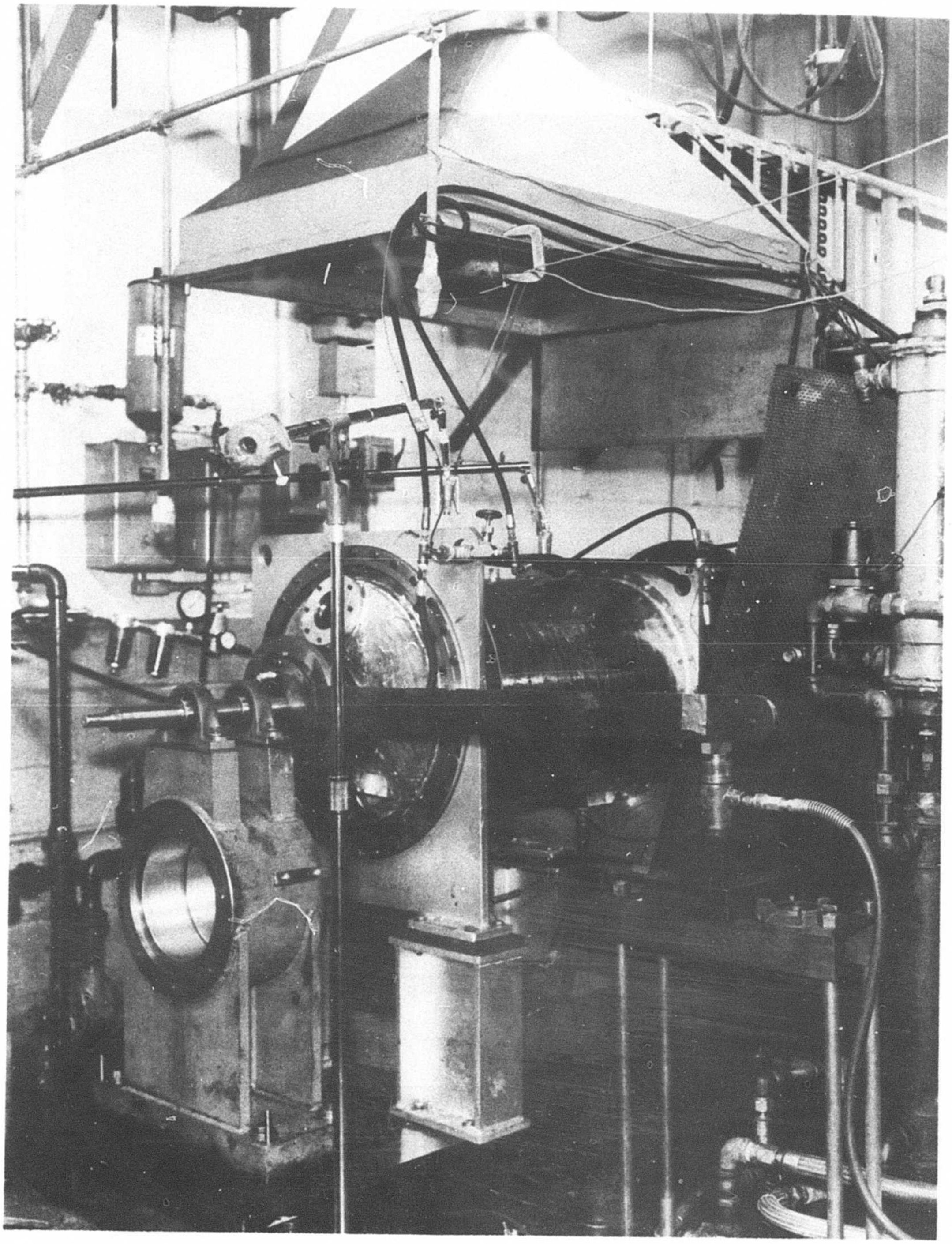


Figure 19. Test Rig on Test Stand.

TEST RESULTS

STATIC

The results of the static tests on the FP500 and FP501 gearbox indicate good load distribution between the planet spindles and gear tooth load patterns.

The results of the FP500 planet spindle load-sharing evaluation are shown in Tables III and V. Table III represents data taken from the strain gauge bridge on each individual quill shaft of the planet spindles. This shows that at 50% of rated power and above, the torque variations are less than 10%. The data taken from the output ring gear, Table V, shows essentially the same results. It is felt that the data obtained from the individual planet quill shafts is more representative since the sensitivity of this system is much higher.

Tables IV and VI show the same data for the FP501 gearbox. It can be seen that the stiffer system of FP501 results in a wider spread of load distribution at lower loads. However, at the important high load points this system is comparable to the FP500 configuration.

The results of the gear tooth load pattern evaluation of the FP500 and FP501 are shown pictorially by Figures 20 and 21. These figures show the "lifted" prints or replicas of the actual pattern for each of the three gear meshes in the transmission. The gear meshing patterns are shown, in the pictures, by the light areas where the red lead was removed during rotation under load. There are charts for 25, 50, 75, 100 and 125% of rated torque. As can be seen from these charts, there is full face contact for each condition, with the contact being essentially even. This represents the condition required for the free planet concept to be successful. It shows a lack of corner loading and also verifies the self-aligning hypothesis of the concept.

TABLE III. PLANET SPINDLE TORQUE VARIATION FP500 BASED ON SPINDLE MEASUREMENTS

Power Setting As % Of Rated	% Torque Variation
25	16.8
50	10.0
75	8.0
100	9.8
125	9.2

TABLE IV. PLANET SPINDLE TORQUE VARIATION FP501 BASED ON SPINDLE MEASUREMENTS

Power Setting As % Of Rated	% Torque Variation
25	26.8
50	17.7
75	12.2
100	10.8
125	8.6

TABLE V. PLANET SPINDLE TORQUE VARIATION FP500 BASED ON RING GEAR MEASUREMENTS	
Power Setting As % Of Rated	% Torque Variation
25	20.8
50	12.2
75	8.4
100	7.5
125	9.0

TABLE VI. PLANET SPINDLE TORQUE VARIATION FP501 BASED ON RING GEAR MEASUREMENTS	
Power Setting As % of Rated	% Torque Variation
25	27.5
50	14.1
75	15.3
100	9.2
125	9.0

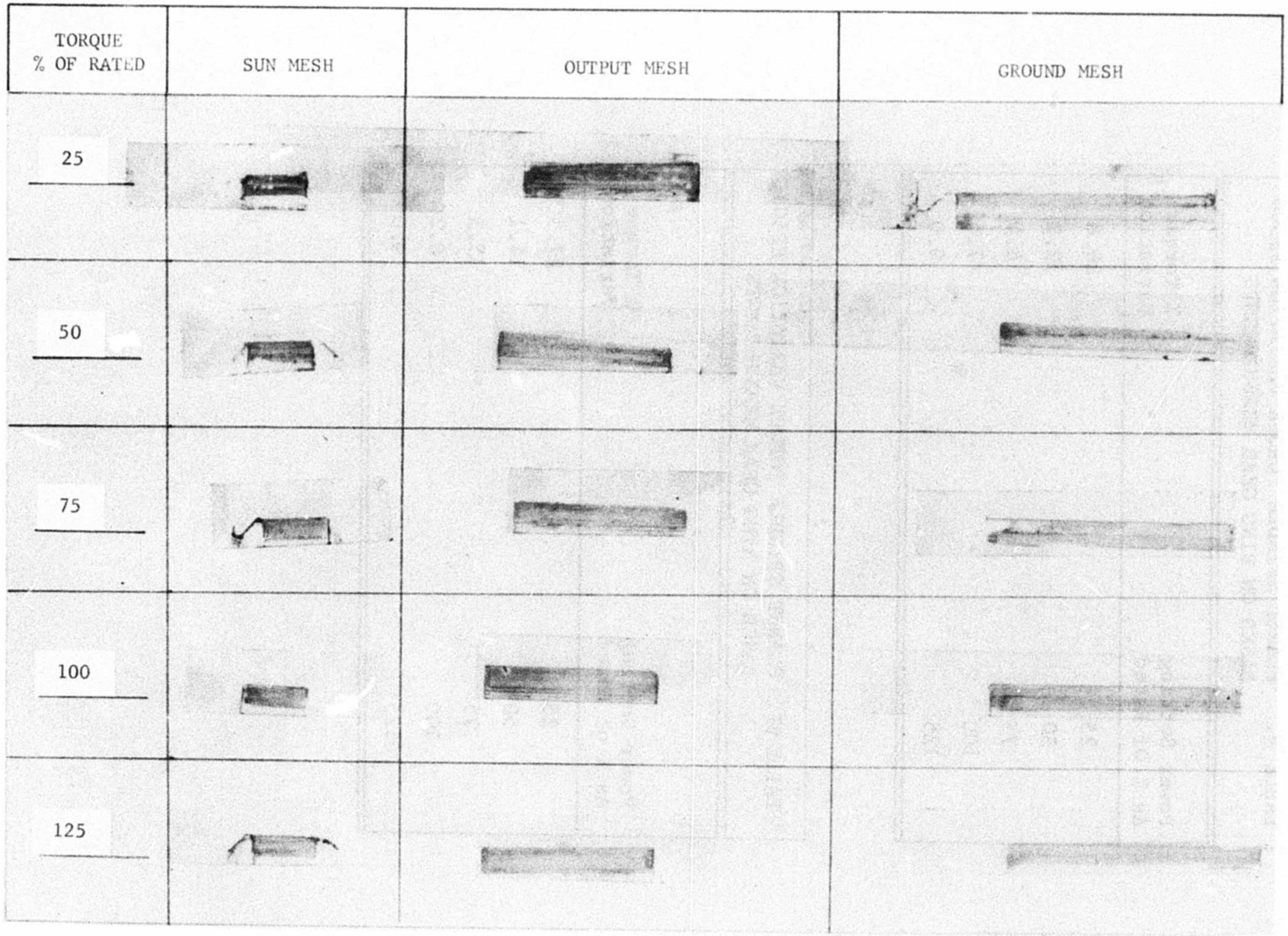


Figure 20. Static Evaluation - FP500 Gear Tooth Load Patterns.

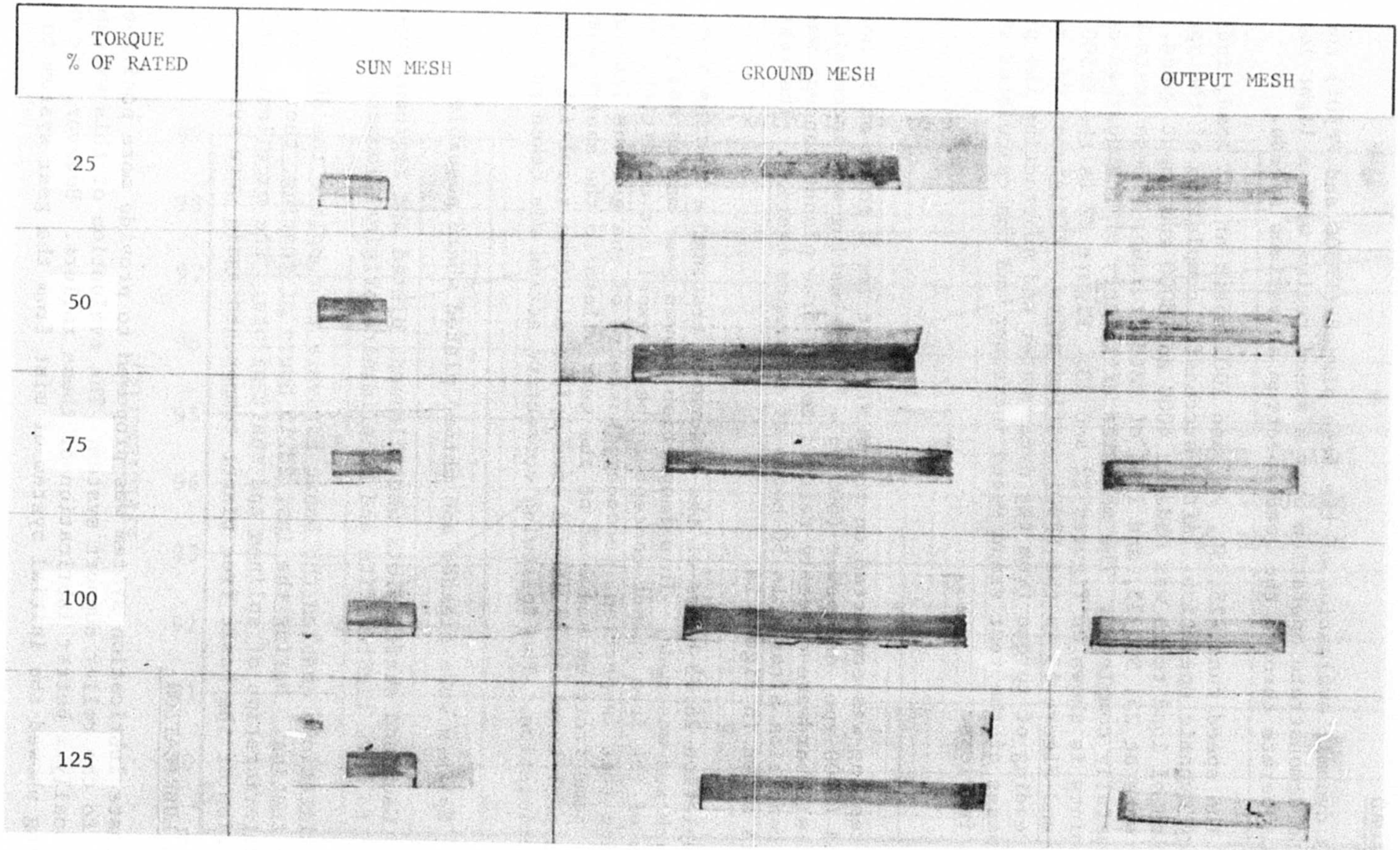


Figure 21. Static Evaluation - FP501 Gear Tooth Load Patterns.

SPEED AND LOAD

Preliminary dynamic evaluation of the free planet (FP500 and FP501) concepts was to demonstrate operation over a speed regime with a light load and then to operate through the load spectrum at various speeds.

Results of the speed runs, 25, 50, 75, and 100% speed at 10% load, indicated satisfactory dynamic operation. After successful completion of the speed runs a series of load tests was made at 4000 and 8000 rpm input speed. These runs were at 25, 50, 75, and 100% of rated torque. These tests were also successfully completed. The measured efficiency of the gearbox at each of these points is shown on Figures 22 and 23. Figure 22 is the FP500 configuration and Figure 23 is the FP501 configuration. Efficiency was measured by direct reading of torque from the dynamometer and subtracting the premeasured losses of the test stand speed increaser and the calculated rig thrust bearing loss.

ENDURANCE

Endurance testing was conducted on the FP500 unit for a total of 50 hours at rated speed (8000 rpm) and power (500 hp). This testing was successfully completed; all hardware directly related to the free planet concept was in excellent condition after this 50 hours of endurance testing. The test hardware is shown in Figure 24.

At the completion 26.75 hours of the endurance testing, a routine inspection of the gearboxes was made. This inspection showed gear meshes and primary components of the free planet concept to be in excellent condition. However, during this inspection, secondary hardware of the gearbox indicated distress. Some fretting and wear of the gear pilots to the shafts was found. Further examination showed some spline distress resulting from pilot deterioration. The following corrective action was taken on these parts:

1. Splines were cleaned and chrome plated where possible.

2. Gear and shaft pilots were cleaned, plated and reground oversize to give tighter fits and to eliminate original looseness.

These modifications enabled the basic hardware to complete the 50 hours of endurance testing. During the last 23.15 hours of testing, there was no further deterioration of splines and shaft pilots. The excellent performance and condition of the basic free planet components again were evident.

ALTERNATE LUBRICATION

The alternate lubrication system was proposed to provide more positive lubrication to the roller support system. The evaluation of this system showed visually a better lubrication of these rollers. However, the original testing showed the initial system of mist from the gear system to be adequate.

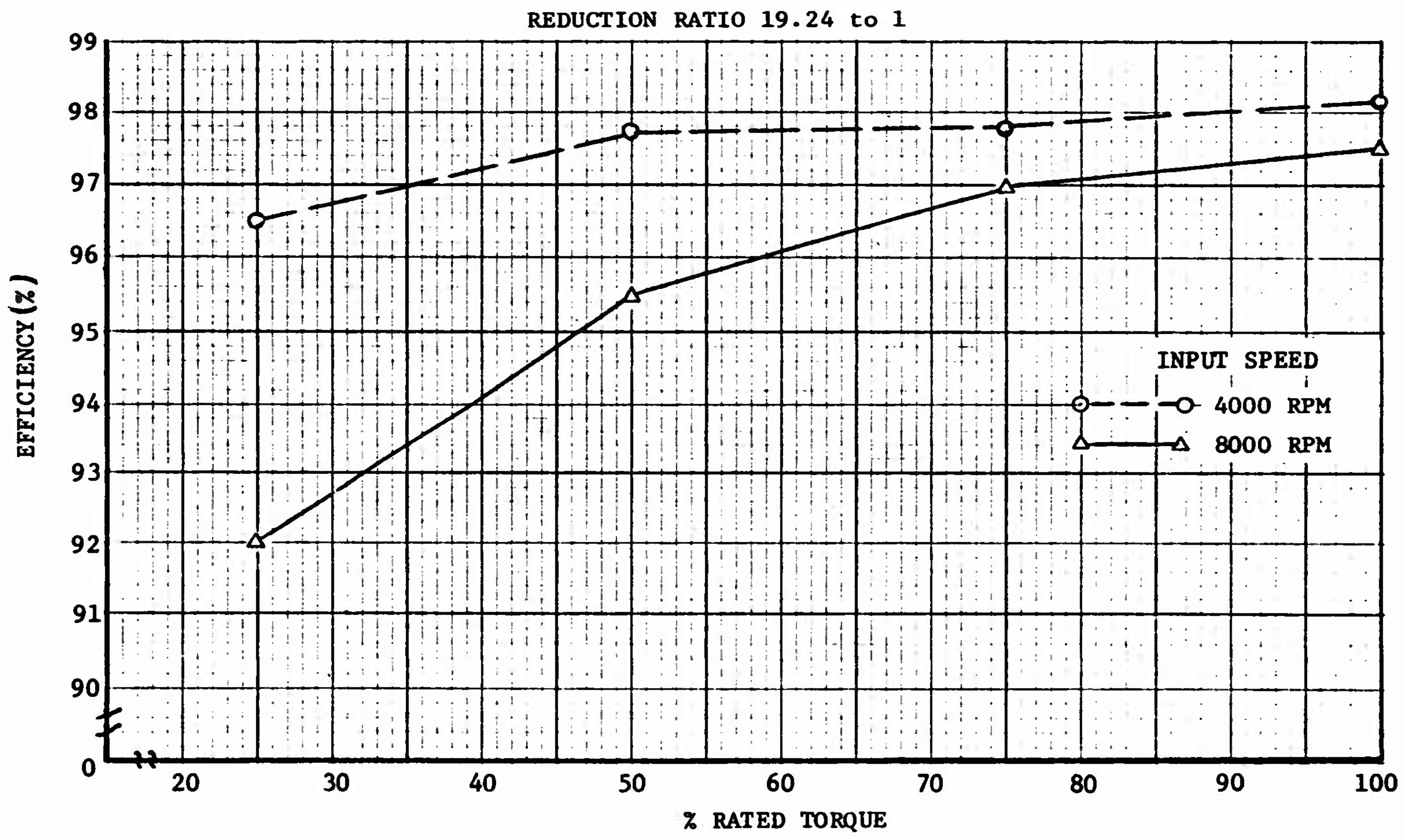


Figure 22. FP500 Efficiency.

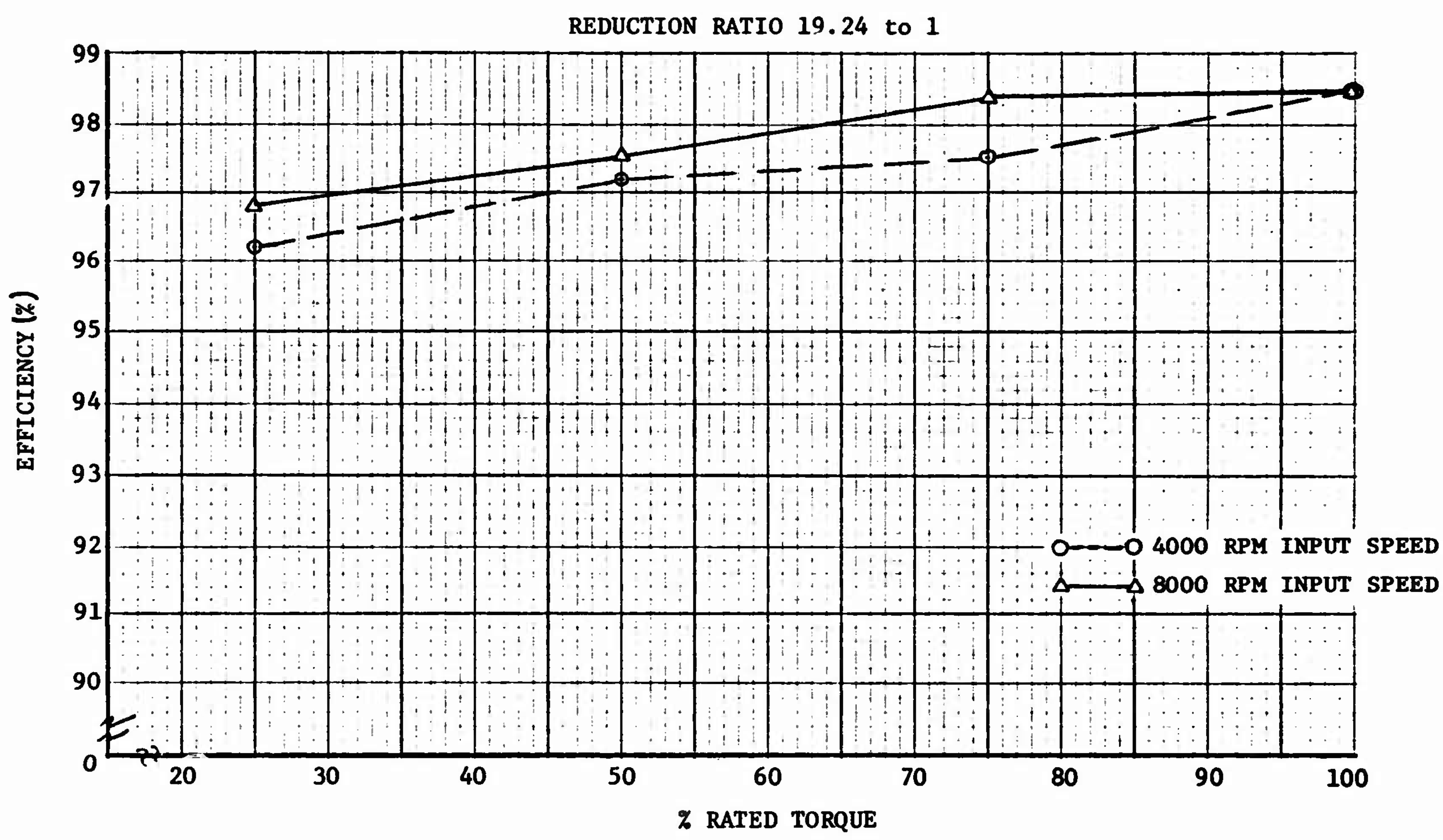


Figure 23. PF501 Efficiency

The evaluation of the alternate lubrication system was conducted by re-running the load tests used for basic evaluation of the units, namely, 4000 rpm at 25, 50, 75, and 100% of rated torque and 8000 rpm at 25, 50, 75, and 100% of rated torque.

OVERSPEED AND OVERLOAD

The dynamic capability of the free planet concept has also been demonstrated at overload and overspeed conditions. Testing was successfully completed on the FP501 unit at the following conditions:

1. 125% rated torque and rated speed.
2. 100% rated torque and 125% rated speed.
3. 125% rated torque and 125% rated speed.

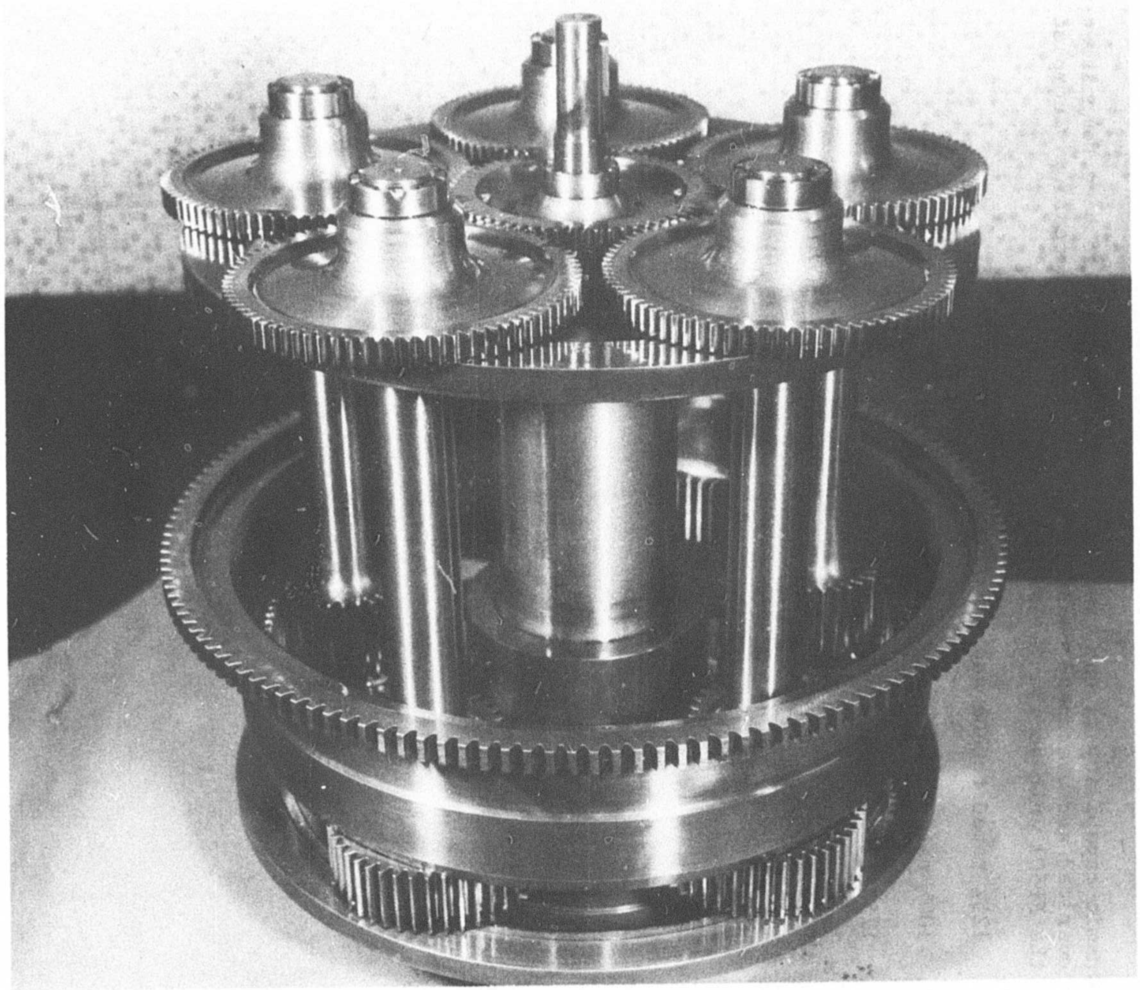


Figure 24. Free Planet Assembly After 50-Hour Endurance Test.

EVALUATION AND ANALYSIS OF DESIGN AND TESTS

WEIGHT COMPARISONS

Preliminary design studies indicate that the free planet transmission concept (FPT) offers significant weight advantages over comparable conventional current-state-of-the-art helicopter transmissions. This analysis included a comparison of helicopter transmissions ranging from approximately 1200-horsepower (Reference 1) to 5000-horsepower (Reference 2). Similar gear materials and stress levels were used for this comparison. Figure 25 was used as a base to depict the results of one study in Reference 1. Here the weight of various existing transmission systems is plotted against the output torque. The basic curve was extracted from Reference 3. The two new points superimposed on this curve (designated as point A and point B) depict the results of this preliminary design and weight analysis study. Point A represents a transmission rated at an output torque of approximately 20,000 ft-lb (1200-horsepower) and point B was rated at about 166,000 ft-lb 5000-horsepower). A weight reduction of 70 lb can be seen for transmission A, and a weight reduction of approximately 1000 lb can be seen for transmission B. These weight savings, when converted to percentages, range from approximately 20% to over 50%.

This type of preliminary design and weight analysis is valid in determining weight trends and gross magnitudes.

It is felt that another approach to a transmission weight comparison is a qualitative analysis. This technuque consists of examining and counting all major weight components of the two systems being compared. It is assumed that the gear tooth loading and the geometry in both transmission systems are similar, and the technique relies on the number of major weight elements required to functionally perform the same job.

In this connection, the free planetary transmission can be grouped into the planetary gear family. When a comparison of all major planetary transmission components is made with those of a free planetary, it is noted that the free planet concept does not have a carrier, does not have planet bearings, and does not require the same type of conventional structural housing. The free planet employs similar types of gears as a conventional planetary does, employs two ring gears, and utilizes radial support rings. Table VII compares a conventional planetary gear with a reduction ratio of 20:1 versus a functionally comparable free planet transmission.

This analysis shows a reduction of approximately 20% in terms of numbers of major components.

Perhaps a more meaningful comparison of this new transmission system can be made in terms of reliability and maintainability.

A review of the current literature on reliability and maintainability of existing helicopter transmissions indicates that antifriction bearing elements represent a major R&M problem area. If these elements could be completely eliminated or at least their usage substantially reduced, the R&M

TABLE VII. QUANTITATIVE ANALYSIS TRANSMISSION COMPARISON

Major Components	Conventional Two Stage Planetary	Free Planet Transmission
Housing	1	0*
Carrier	2	0
Gears	14	18
Planet Bearings	10	0
Ring Gears	2	2
Support Rings	0	4
Total	29	24

* Does not require a structural housing. Attachment to the structure can be accomplished via the fixed ring gear.

problems of helicopter transmission would be significantly reduced. An elimination of all planet bearings represents a significant reduction in the total number of antifriction bearings used in a conventional helicopter planetary transmission system. This characteristic is ranked as one of the more striking features of the free planetary transmission design.

EFFICIENCY

The efficiency achieved by these demonstration units appears to be comparable to that achieved with conventional planetary designs of this reduction ratio.

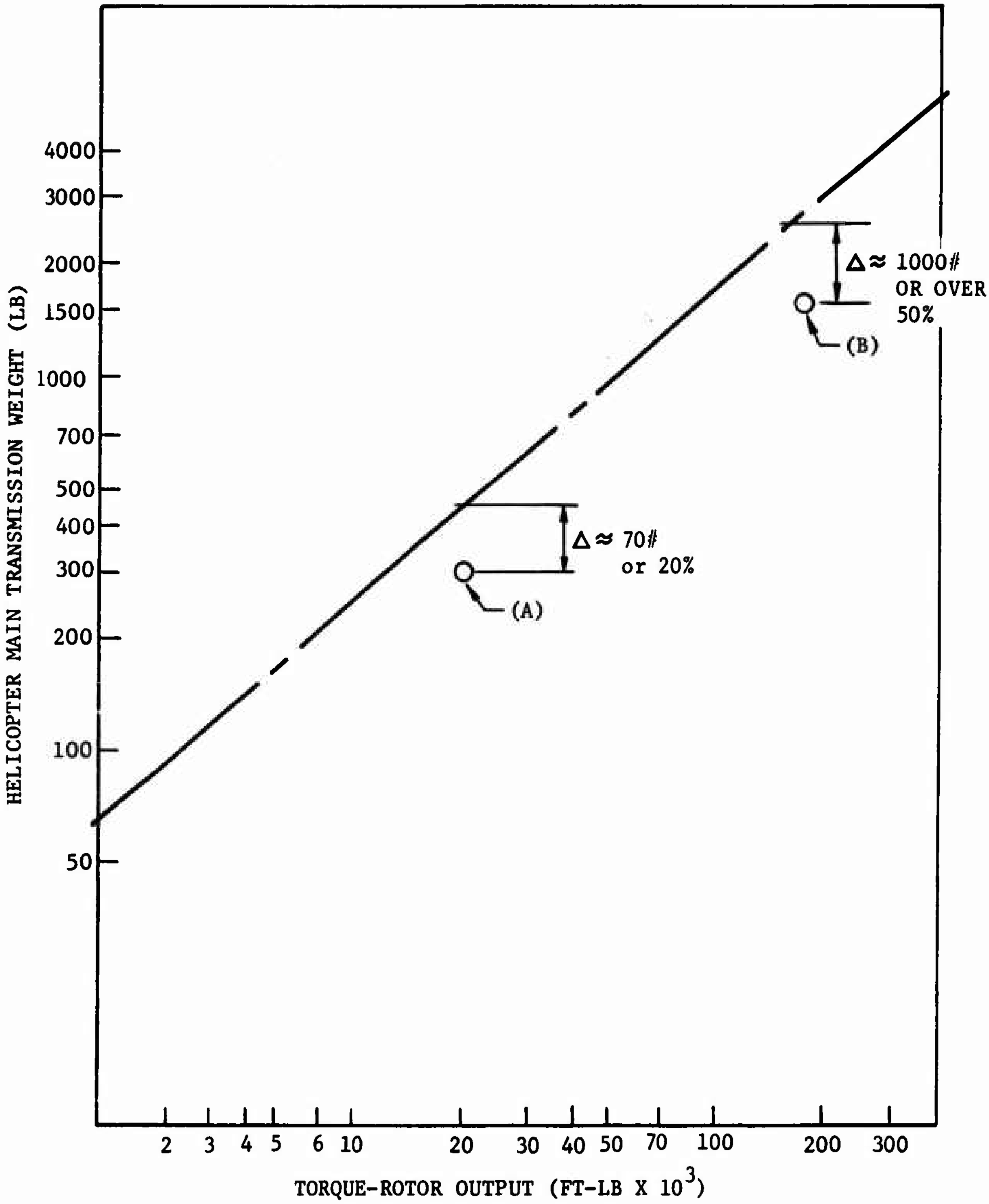


Figure 25. Helicopter Main Transmission Weight Vs Torque-Rotor Output.

CONCLUSIONS

1. On the basis of the results achieved with the feasibility demonstration program, the free planet type transmission is a viable and competitive concept.

2. The force balance principle of the free planet concept appears to be sound both statically and dynamically.

RECOMMENDATIONS

Based on the favorable results obtained from the current feasibility demonstration program, it is recommended that the following areas be further explored:

1. Demonstrate endurance capability of the FP501 type configuration.

2. Define to a greater degree the dynamic behavior of the concept.

3. Conduct preliminary designs of helicopter transmissions to provide complete comparisons of current technology to the free planet concept.

4. Use the results of these preliminary designs as a basis to plan further test evaluations.

LITERATURE CITED

1. Curtiss-Wright Corporation Report No. C-3111 titled "Comparison Summary of High Ratio Free Planet Transmission for UH-1 Helicopter".

2. Lastine, James L., and White, Graham, "Advanced Technology VTOL Drive Train Configuration Study," Sikorsky Aircraft; USSAAVLABS Technical Report 69-69, U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia, January 1970, AD867905.

3. NAEC-AEL-1849 Report titled "State of the Art Review on Helicopter Transmission, Turboprop Gearboxes and Lubrication Thereof".

APPENDIX
DESIGN OF FREE PLANET DEMONSTRATION HARDWARE

DESIGN OF FREE PLANET TRANSMISSION

The free planet transmission concept covers broadly those planetary arrangements wherein the planets are not constrained by being mounted on a carrier or spider.

The configuration consists of an input sun gear meshing with five planet gears, a fixed internal gear, and an output internal gear.

The planet gear assembly is compounded and consists of three gears. The first plane of the planet gears meshes with the sun gear, the second plane meshes with the output internal ring gear, and the third plane meshes with a stationary internal ring gear. The first and second planes of the gear are splined, double piloted, and locked to a quill shaft by a nut and cup lock. The third plane is splined, double piloted, and locked to the second and third plane planet gears have a tooth in line at the top vertical centerline and the first plane planet gear has a tooth in line 180° at the bottom vertical centerline.

The planetary gear train tooth numbers have been selected so that the configuration is hunting and nonfactorizing.

The center distance of the orbiting planets is controlled with three rings: two external rings and one internal ring. The rings are located in a plane outside the first plane of planets (sun gear mesh) and outside the third plane of planets (stationary ring gear). The fore and aft shoulders retaining the O.D. and I.D. rings have a 4-to 6-minute taper for lubrication of the rings.

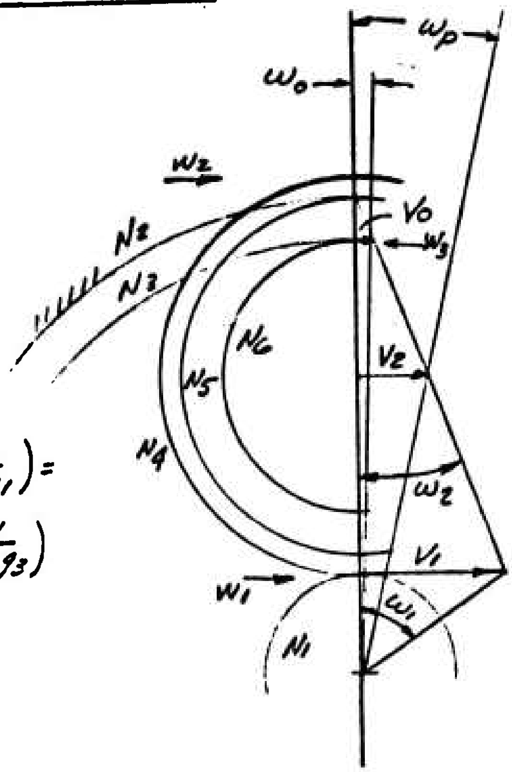
Since the feasibility design is to be tested in a Hopkinson rig (back-to-back test rig arrangement), the planets are restrained axially by a ball thrust bearing attached to the inner ring located aft of the third plane of planets. The output internal ring gears are restrained axially by a plain flat face AMS 6415 steel thrust ring running against an AMS 4845 bronze ring.

In the Hopkinson rig configuration, the input sun gears of the back-to-back transmission are connected by a Thomas N0262 Series 50 coupling. The output internal ring gears are connected to each other by a splined drum.

The nonrotating internal ring gears absorb the reaction torque, and one of the nonrotating internal ring gears is attached to the rear housing closure, which in turn is piloted to the main housing.

The rear closure housing is rotated by an actuator to introduce torque into the drive system.

GEAR RATIO GEOMETRY



$$mg_1 = \frac{N_1}{N_4}$$

$$mg_2 = \frac{N_2}{N_5}$$

$$mg_3 = \frac{N_3}{N_6}$$

$$N_1 + N_4 = N_2 - N_5 = N_3 - N_6$$

$$N_4(mg_1 + 1) = N_5(mg_2 - 1) =$$

$$N_6(mg_3 - 1) = N_1\left(1 + \frac{1}{mg_1}\right) =$$

$$N_2\left(1 - \frac{1}{mg_2}\right) = N_3\left(1 - \frac{1}{mg_3}\right)$$

$$V_1 = \omega_1 N_1 \; ; \quad \omega_1 = \frac{V_1}{N_1}$$

$$V_2 = \omega_p (N_1 + N_4)$$

$$\omega_p = \frac{V_2}{N_1 + N_4}$$

$$V_2 = \omega_2 N_5 \quad ; \quad \omega_2 = \frac{V_2}{N_5}$$

$$V_0 = \omega_0 N_3 \quad ; \quad \omega_0 = \frac{V_0}{N_3}$$

$$mg = \frac{\omega_1}{\omega_0} = \frac{V_1 N_3}{V_0 N_1}$$

$$\frac{V_1}{N_4 + N_5} = \frac{V_0}{N_5 - N_6}$$

$$V_0 = V_1\left(\frac{N_5 - N_6}{N_4 + N_5}\right)$$

$$mg = \frac{V_1 N_3}{N_1}\left(\frac{N_4 + N_5}{V_1(N_5 - N_6)}\right) = \frac{N_3}{N_1}\left(\frac{N_4 + N_5}{N_5 - N_6}\right)$$

$$mg = \frac{\dfrac{N_4}{N_1} + \dfrac{N_5}{N_1}}{\dfrac{N_5}{N_3} - \dfrac{N_6}{N_3}}$$

$$\frac{N_4}{N_1} = \frac{1}{mg_1}$$

$$\frac{N_5}{N_1} = \frac{1 + \frac{1}{mg_1}}{mg_2 - 1} = \frac{1 - \frac{1}{mg_3}}{mg_2 - 1}$$

$$\frac{N_6}{N_3} = \frac{1}{mg_3}$$

$$mg = \frac{\frac{1}{mg_1} + \frac{mg_1 + 1}{mg_1(mg_2 - 1)}}{\frac{mg_3 - 1}{mg_3(mg_2 - 1)} - \frac{1}{mg_3}}$$

$$mg = \frac{\frac{(mg_2 - 1) + (mg_1 + 1)}{mg_1\cancel{(mg_2 - 1)}}}{\frac{(mg_3 - 1) - (mg_2 - 1)}{mg_3\cancel{(mg_2 - 1)}}}$$

$$mg = \frac{mg_3(mg_2 + mg_1)}{mg_1(mg_3 - mg_2)}$$

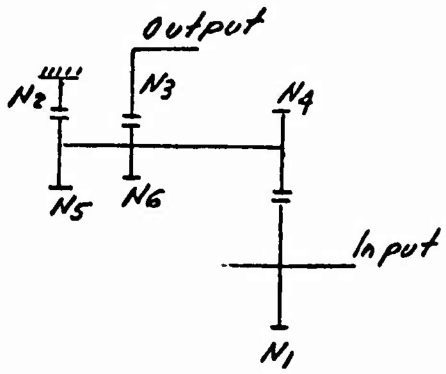
$$mg = \frac{\cancel{mg_3}\left(1 + \frac{mg_2}{mg_1}\right)\cancel{mg_1}}{\cancel{mg_1}\left(1 - \frac{mg_2}{mg_3}\right)\cancel{mg_3}}$$

$$mg = \frac{\left(1 + \frac{mg_2}{mg_1}\right)}{\left(1 - \frac{mg_2}{mg_3}\right)}$$

$$\boxed{mg = \frac{\left(1 + \frac{N_2 N_4}{N_5 N_1}\right)}{\left(1 - \frac{N_2 N_6}{N_5 N_3}\right)}}$$

FINAL RATIO FP 500 AND 501

Given Data



8000 RPM

20:1 Ratio

500 HP

Oil : MIL-L-7808

$N1 = 66T$
$N4 = 79T$

$N3 = 143T$
$N6 = 28T$

$N2 = 158T$
$N5 = 43T$

$$mg = \frac{\left(1 + \frac{N_2 N_4}{N_5 N_1}\right)}{\left(1 - \frac{N_2 N_6}{N_5 N_3}\right)}$$

$$mg = \frac{\left(1 + \left(\frac{158}{43}\right)\left(\frac{79}{66}\right)\right)}{\left(1 - \left(\frac{158}{43}\right)\left(\frac{28}{143}\right)\right)}$$

$$mg = \underline{\underline{19.2425}}$$

Input torque

$$T = \frac{(HP)(63025)}{N} = \frac{(500)(63025)}{(8000)} = \underline{\underline{3,939 \text{ IN.-LB.}}}$$

Output torque

$$T = (3939)(19.2425) = \underline{\underline{75,797 \text{ IN.-LB.}}}$$

(N_4) Sun Planet Torque/mesh $= \left(\frac{3939}{5}\right)\left(\frac{6.2655}{5.2345}\right) = \underline{\underline{943 \text{ IN.-LB.}}}$

(N_1) Sun Gear Torque/mesh $= \left(\frac{3939}{5}\right) = \underline{\underline{788 \text{ IN.-LB.}}}$

(N_3) Output Gear Torque/mesh $= \left(\frac{75,797}{5}\right) = \underline{\underline{15,159 \text{ IN.-LB.}}}$

(N_6) Output Planet Gear Torque per mesh $= \left(\frac{15159}{7.15}\right)(1.40) = \underline{\underline{2,968 \text{ IN.-LB}}}$

(N_2) Stationary Internal Gear/mesh $= (1819)(7.90) = \underline{\underline{14,371 \text{ IN.-LB.}}}$

(N_5) Stationary Internal planet Gear/mesh $= (1819)(2.15) = \underline{\underline{3,911 \text{ IN.-LB.}}}$

FIRST-STAGE GEAR DESIGN

-:- INPUT DATA -:-

PINION TORK	787.6 IN.LB.
PRESSURE ANGLE	20 DEGREES
HERTZ STRESS	135 KSI
FILLET STRESS	39 KSI
GEAR CLEARANCE FACTOR	0.15
DED. CLEAR. FACTOR	0.25
HOB TIP RADIUS FACTOR	0.3

-:- GEAR DATA -:-

	PINION N1		GEAR N4
NUMBER OF TEETH	66		79
PITCH DIAM.-IN.	5.2345		6.2655
FACE WIDTH-IN.		0.189	
ACTUAL GEAR RATIO		1.197	
CENTER DIST.-IN.		5.75	
ADDENDUM-IN.	0.088		0.087
DEDENDUM-IN.	0.098		0.1
BASIC TOOTH THICK.-IN.	0.1251		0.1241
TIP TOOTH THICK.-IN.	0.056		0.057
STD. GEAR SET B/L-IN.			
FILLET RADIUS-IN.	0.026		0.026
CONTACT RATIO		1.979	
DIAMETRAL PITCH		12.6087	
ROLL ANGLES-DEG.			
OUTSIDE DIAM.	26.21		25.34
HPSTC	20.91		20.9
PITCH DIAM.		20.85	
LPSTC	20.81		20.81
MIN. CONTACT DIAM.	17.11		16.38
TOOTH DRIVING LOAD-LB.		301	
UNIT DRIVING LOAD-LB/IN.		1594	
RADIAL TOOTH LOAD-LB.		110	
DEPTH TO MAX. SHEAR-IN.		0.006	
'J' FACTOR	0.519		0.519
'I' FACTOR		0.0875	
'K' FACTOR		559	

SECOND-STAGE GEAR DESIGN

-:- INPUT DATA -:-

PINION TORK	4335 IN.LB.
PRESSURE ANGLE	20 DEGREES
HERTZ STRESS	135 KSI
FILLET STRESS	31 KSI
GEAR CLEARANCE FACTOR	0.15
DED. CLEAR. FACTOR	0.25
HOB TIP RADIUS FACTOR	0.3

-:- GEAR DATA -:-

	Planet PINION	N5	Sun GEAR	N2 Ring 158 T
NUMBER OF TEETH	43		72	
PITCH DIAM.-IN.	4.3		7.2	
FACE WIDTH-IN.		1.327		
ACTUAL GEAR RATIO		1.6744		
CENTER DIST.-IN.		5.75		
ADDENDUM-IN.	0.113		0.107	
DEDENDUM-IN.	0.122		0.128	
BASIC TOOTH THICK.-IN.	0.1596		0.1546	
TIP TOOTH THICK.-IN.	0.066		0.071	
STD. GEAR SET B/L-IN.				
FILLET RADIUS-IN.	0.034		0.033	
CONTACT RATIO		1.926		
DIAMETRAL PITCH		10.		
ROLL ANGLES-DEG.				
OUTSIDE DIAM.	28.97		25.67	
HPSTC	21.19		21.03	
PITCH DIAM.		20.85		
LPSTC	20.75		20.65	
MIN. CONTACT DIAM.	17.97		16.01	
TOOTH DRIVING LOAD-LB.		2016		
UNIT DRIVING LOAD-LB/IN.		1520		
RADIAL TOOTH LOAD-LB.		734		
DEPTH TO MAX. SHEAR-IN.		0.006		
'J' FACTOR	0.489		0.489	
'I' FACTOR		0.1		
'K' FACTOR		564		

SECOND-STAGE GEAR DESIGN

-:- INPUT DATA -:-

PINION TORK	4335 IN.LB.
PRESSURE ANGLE	20 DEGREES
HERTZ STRESS	135 KSI
FILLET STRESS	31 KSI
GEAR CLEARANCE FACTOR	0.15
DED. CLEAR. FACTOR	0.25
HOB TIP RADIUS FACTOR	0.3

-:- GEAR DATA -:-	N5 Planet PINION		Sun GEAR	N2 Ring 158 T
NUMBER OF TEETH	43		72	
PITCH DIAM.-IN.	4.3		7.2	
FACE WIDTH-IN		1.327		
ACTUAL GEAR RATIO		1.6744		
CENTER DIST.-IN.		5.75		
ADDENDUM-IN.	0.113		0.107	
DEDENDUM-IN.	0.122		0.128	
BASIC TOOTH THICK.-IN.	0.1596		0.1546	
TIP TOOTH THICK.-IN.	0.066		0.071	
STD. GEAR SET B/L-IN.				
FILLET RADIUS-IN.	0.034		0.033	
CONTACT RATIO		1.926		
DIAMETRAL PITCH		10.		
ROLL ANGLES-DEG.				
OUTSIDE DIAM.	28.97		25.67	
HPSTC	21.19		21.03	
PITCH DIAM.		20.85		
LPSTC	20.75		20.65	
MIN. CONTACT DIAM.	17.97		16.01	
TOOTH DRIVING LOAD-LB.		2016		
UNIT DRIVING LOAD-LB/IN.		1520		
RADIAL TOOTH LOAD-LB.		734		
DEPTH TO MAX. SHEAR-IN.		0.006		
'J' FACTOR	0.489		0.489	
'I' FACTOR		0.1		
'K' FACTOR		564		

SECOND-STAGE PLANETARY RING GEAR MESH

	PLANET		RING GEAR
NUMBER OF TEETH	43		158
PITCH DIAM.-IN.	4.3		15.8
ROLLING DIAM.-IN.	4.3		15.8
INSIDE DIAM.-IN.			15.614
ROOT DIAM.-IN.			16.057
TRANSV. PRESS. ANGLE-DEG.			
AT PITCH DIAM.		20.	
AT ROLLING DIAM.		20.	
CENTER DIST.-IN.		5.75	
TOOTH THICK.,BASIC-IN.			
TRANSV. AT ROLL. DIAM.	0.1596		0.1546
REFERRED TO PITCH DIAM.			
ADDENDUM-IN.	0.113		0.093
DEDENDUM-IN.			0.128
REFERRED TO ROLL. DIAM.			
TRANSV. PITCH		10.	
ADDENDUM-IN.	0.113		0.093
DEDENDUM-IN.			0.128
FILLET RAD.-IN.			0.042
HERTZ STRESS RATIO		0.675	
APPROACH RATIO			0.5
TOOTH DRIVING LOAD-LB.		2016	
TOOTH RADIAL LOAD-LB.		734	
INVOLUTE ROLL ANGLES-DEG.			
INSIDE DIAM.			18.66
LPSTC			20.77
ROLLING P.D.		20.85	
HPSTC			20.94
CONTACT DIA.			23.05

THIRD-STAGE GEAR DESIGN

-:- INPUT DATA -:-

PINION TORK	3247 IN.LB.
PRESSURE ANGLE	20 DEGREES
HERTZ STRESS	135 KSI
FILLET STRESS	26.5 KSI
GEAR CLEARANCE FACTOR	0.15
DED. CLEAR. FACTOR	0.25
HOB TIP RADIUS FACTOR	0.3

Ring
N3
143 T

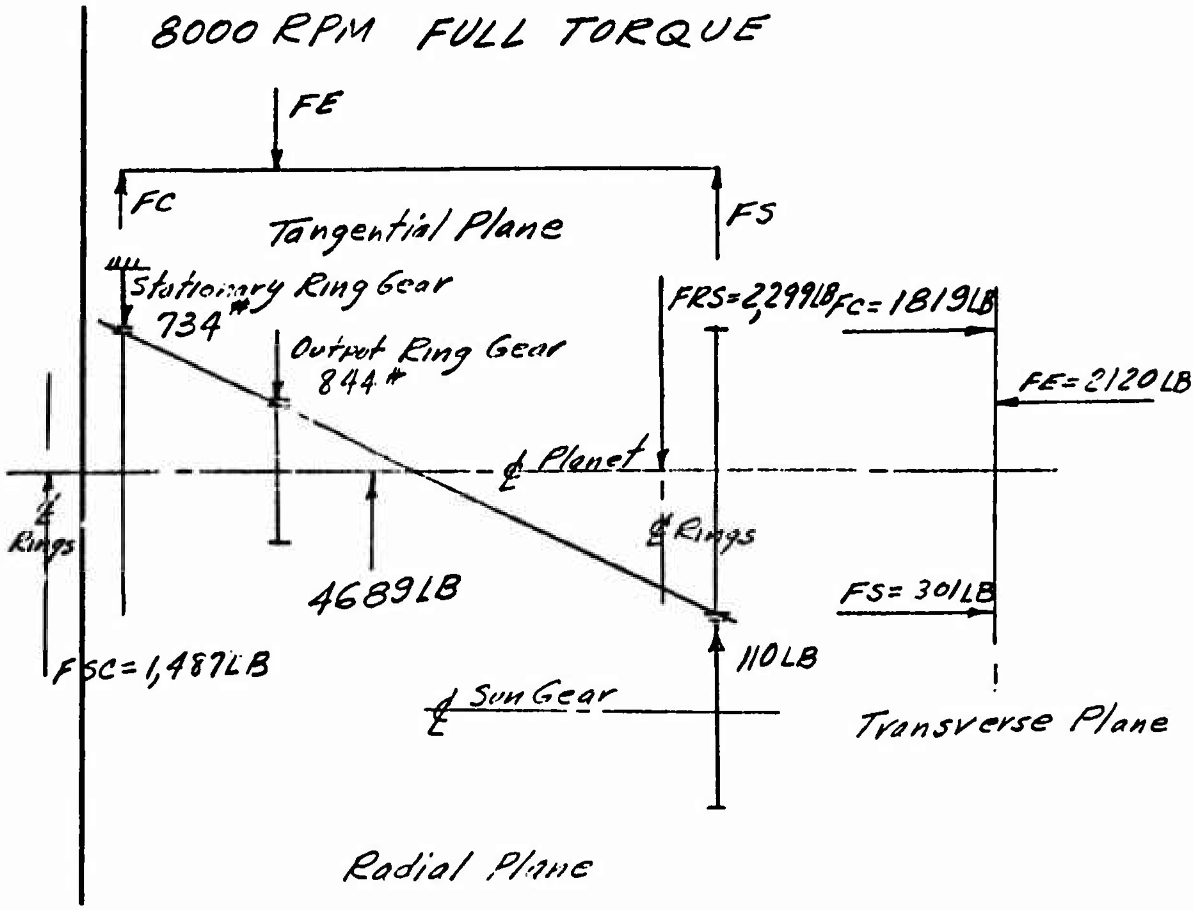
-:- GEAR DATA -:-

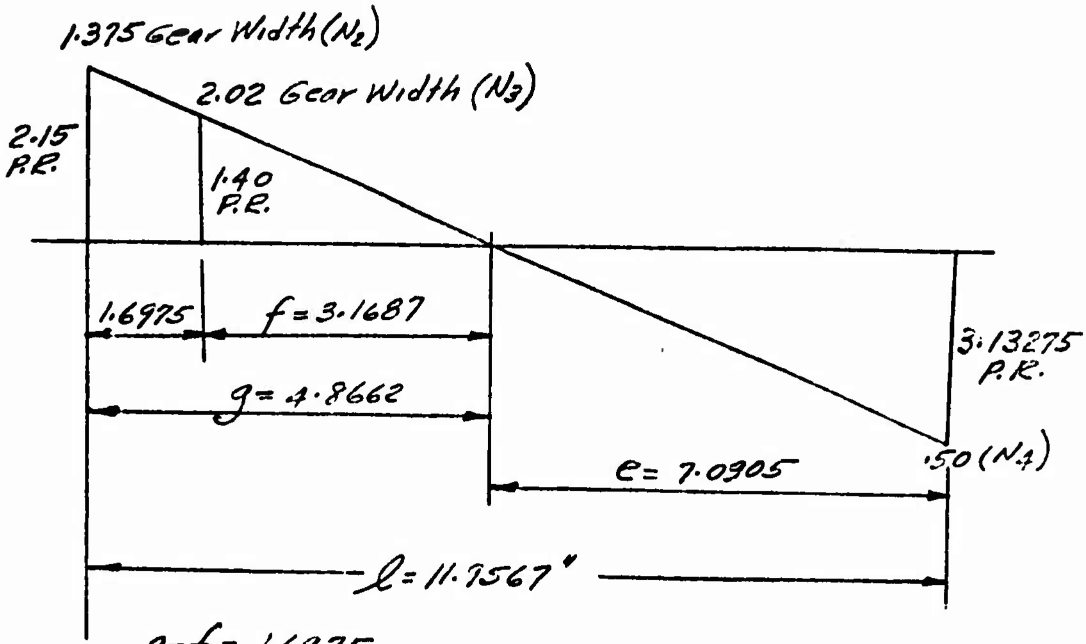
	Planet PINION N6		Sun GEAR
NUMBER OF TEETH	28		87
PITCH DIAM.-IN.	2.8		8.7
FACE WIDTH-IN.		1.866	
ACTUAL GEAR RATIO		3.1071	
CENTER DIST.-IN.		5.75	
ADDENDUM-IN.	0.119		0.101
DEDENDUM-IN.	0.116		0.134
BASIC TOOTH THICK.-IN.	0.1634		0.1508
TIP TOOTH THICK.-IN.	0.06		0.073
STD. GEAR SET B/L-IN.			
FILLET RADIUS-IN.	0.035		0.032
CONTACT RATIO		1.876	
DIAMETRAL PITCH		10.	
ROLL ANGLES-DEG.			
OUTSIDE DIAM.	33.12		24.73
HPSTC	21.77		21.08
PITCH DIAM.		20.85	
LPSTC	20.78		20.56
MIN. CONTACT DIAM.	19.61		16.91
TOOTH DRIVING LOAD-LB.		2319	
UNIT DRIVING LOAD-LB/IN.		1243	
RADIAL TOOTH LOAD-LB.		844	
DEPTH TO MAX. SHEAR-IN.		0.005	
'J' FACTOR	0.466		0.466
'I' FACTOR		0.1188	
'K' FACTOR		587	

THIRD-STAGE PLANETARY RING GEAR MESH

	PLANET		RING GEAR
NUMBER OF TEETH	28		143
PITCH DIAM.-IN.	2.8		14.3
ROLLING DIAM.-IN.	2.8		14.3
INSIDE DIAM.-IN.			14.122
ROOT DIAM.-IN.			14.567
TRANSV. PRESS. ANGLE-DEG.			
AT PITCH DIAM.		20.	
AT ROLLING DIAM.		20.	
CENTER DIST.-IN.		5.75	
TOOTH THICK.,BASIC-IN.			
TRANSV. AT ROLL. DIAM.	0.1634		0.1508
REFERRED TO PITCH DIAM.			
ADDENDUM-IN.	0.119		0.089
DEDENDUM-IN.			0.134
REFERRED TO ROLL. DIAM.			
TRANSV. PITCH		10.	
ADDENDUM-IN.	0.119		0.089
DEDENDUM-IN.			0.134
FILLET RAD.-IN.			0.042
HERTZ STRESS RATIO		0.78	
APPROACH RATIO			0.494
TOOTH DRIVING LOAD-LB.		2331	
TOOTH RADIAL LOAD-LB.		848	
INVOLUTE ROLL ANGLES-DEG.			
INSIDE DIAM.			18.52
LPSTC			20.73
ROLLING P.D.		20.85	
HPSTC			21.04
CONTACT DIA.			23.25

FORCE DIAGRAM FP 500 AND 501





$$g - f = 1.6975$$

$$g = 1.6975 + f$$

$$\frac{1.40}{f} = \frac{2.15}{g} = \frac{2.15}{1.6975 + f}$$

$$1.40f + 2.3765 = 2.15f$$

$$f = \frac{2.3765}{.75} = 3.1686666$$

$$g = 4.8662$$

$$\frac{3.13275}{e} = \frac{2.15}{4.8662}$$

$$e = \frac{(3.13275)(4.8662)}{2.15} = 7.0905$$

$$\ell = e + g = 7.0905 + 4.8662$$

$$\ell = \underline{11.9567 \text{ IN.}}$$

FREE PLANET - MESHING REQUIREMENTS

$$J_1 = \frac{i N_2 + N_1}{n}$$

i = integer from 1 to n
(i must be the same for all three Meshes)
N_2 = Planet Gear
N_1 = Mating Gear with Planet
(+ external)
(− internal)

$$J_1 = \frac{i(28) - 143}{5} = \frac{28 - 143}{5} = -\frac{115}{5} = -25$$

$$J_2 = \frac{i(43) - 158}{5} = \frac{43 - 158}{5} = -\frac{115}{5} = -25$$

$$J_3 = \frac{i(79) + 66}{5} = \frac{79 + 66}{5} = \frac{145}{5} = +29$$

Three planet Gear Meshes as follows. (Ref Sheet 2)

#1 Planet
43T – Top tooth on ℄
28T – Top tooth on ℄
79T – Tooth on bottom ℄
all teeth Line up on Vertical ℄

#2 Planet – Rotate 72° from radial ℄ in CCW rotation

$$\frac{72}{360}(43) = \frac{3096}{360} = 8.600$$

$$\frac{72}{360}(28) = \frac{2016}{360} = 5.600$$

$$\frac{72}{360}(79) = \frac{5688}{360} = 15.800$$

Planet #3 Rotate 144° from radial $\mathbb{C}$ in CCW Rotation.

$$\left.\begin{aligned}
&\frac{144}{360}(43) = \frac{6192}{360} = 17.200\\
&\frac{144}{360}(28) = 11.200\\
&\frac{144}{360}(79) = 31.600
\end{aligned}\right\}$$

Planet #4 Rotate 216° from radial $\mathbb{C}$ in CCW Rotation

$$\left.\begin{aligned}
&\frac{216}{360}(43) = \frac{9288}{360} = 25.800\\
&\frac{216}{360}(28) = 16.800\\
&\frac{216}{360}(79) = 47.400
\end{aligned}\right\}$$

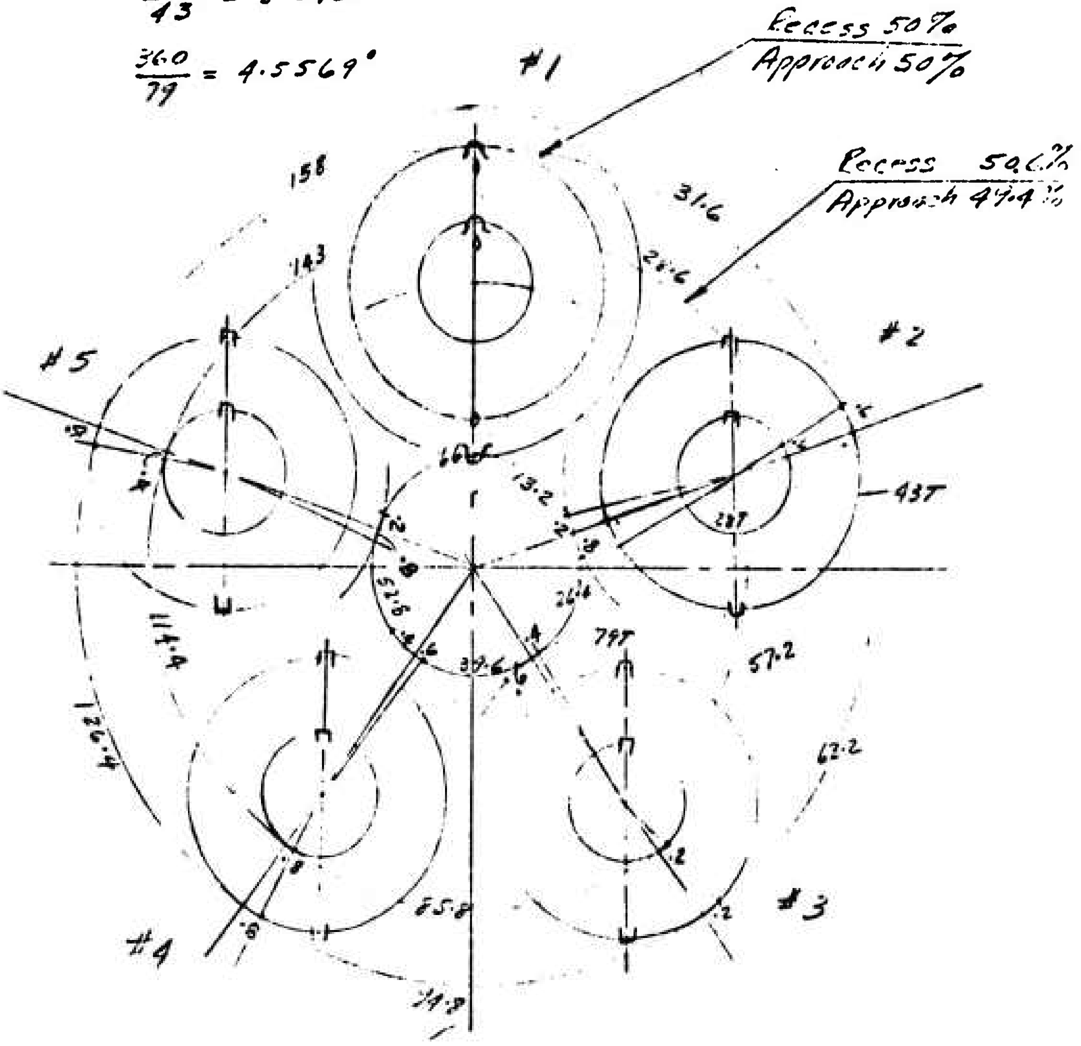
Planet #5 Rotate 288° from radial $\mathbb{C}$ in CCW Rotation

$$\left.\begin{aligned}
&\frac{288}{360}(43) = \frac{12384}{360} = 34.400\\
&\frac{(288)(28)}{360} = 22.400\\
&\frac{(288)(79)}{360} = 63.200
\end{aligned}\right\}$$

Cir Pitch $\frac{360}{28} = 12.8571°$

$\frac{360}{43} = 8.3720°$

$\frac{360}{79} = 4.5569°$



FREE PLANET GEAR DATA FP 500 AND 501

RESULTS, SKETCHES & FORMULAS

	1st Sun	1st Planet	2nd Planet	2nd Ring	3rd Planet	3rd Ring
N	66	79	43	158	28	143
P	12.6087	12.6087	10.00	10.00	10.00	10.00
ϕ	20°	20°	20°	20°	20°	20°
D_P	5.2345	6.2655	4.3000	15.8000	2.8000	14.3000
C.D.	5.75	5.75	5.75	5.75	5.75	5.75
R.O.	.001	.001	.001	.0015	.001	.0015
T_1	.1251	.1241	.1596	.1546	.1634	.1508
Mach T_1	.1221 / .1201	.1211 / .1191	.1566 / .1546	.1526 / .1486	.1609 / .1589	.1488 / .1448
BL	.003-.005	.003-.005	.003-.005	.002-.006	.003-.005	.002-.006
P_o	.2341	.2341	.2952	.2952	.2952	.2952
D_b	4.9188	5.8877	4.0407	14.8471	2.6311	13.4376
d_m	.1400	.1400	.1728	.1745	.1728	.1800
M	5.4273 / 5.4324	6.4548 / 6.4599	4.5334 / 4.5383	15.5694 / 15.5581	3.0438 / 3.0485	14.0571 / 14.0457
D_o	5.410 / 5.400	6.440 / 6.430	4.526 / 4.516	15.629 / 15.614	3.038 / 3.028	14.137 / 14.122
D_R	5.039 / 5.014	6.066 / 6.041	4.056 / 4.031	16.091 / 16.056	2.568 / 2.543	14.603 / 14.568
V_f	.026	.026	.034	.042	.035	.042
θ_o	26.21	25.34	28.97	18.66	33.12	18.52
θ_h	20.91	20.90	21.19	20.77	21.77	20.73
θ	20.85	20.85	20.85	20.85	20.85	20.85
θ_l	20.81	20.81	20.75	20.94	20.78	21.04
θ_c	17.11	16.38	17.97	23.05	19.61	23.25
Pitch Tol	.0004	.0004	.0004	.0004	.0004	.0004
Prof. Tol	.0007	.0007	.0007	.0008	.0007	.0008
Lead	.0004	.0004	.0006	.0006	.0005	.0005

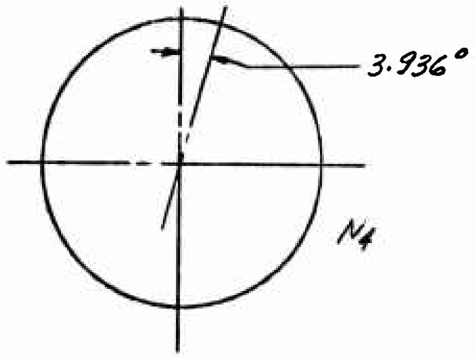
PLANET TORSIONAL TWIST FP 500

Torque = 943 IN# (Sun Planet Torque/mesh)

$$\theta = \frac{2Tl}{\pi r^4 G} = \frac{(2)(943)(11.625)}{(\pi)(.31)^4(11\times10^6)} = .06869 \text{ radians}$$

$$\theta^\circ = (.06869)(57.3) = 3.936^\circ$$

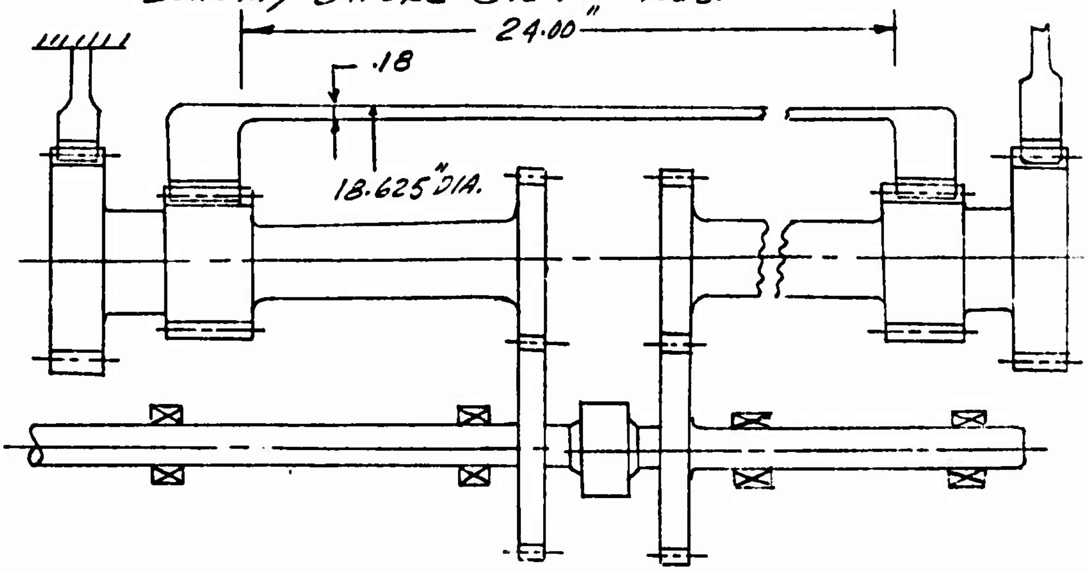
Stroke $(R\theta) = (.06869)(2.0774) = .14273''$ for N_4



$$S_{max} = \frac{2T}{\pi r^3} = \frac{(2)(943)}{(\pi)(.31)^3} = \underline{20,151 \text{ P.S.I.}}$$

Quill Shaft Stress

Loading Stroke @ 12.75" Rad.



$$\theta = \frac{2Tl}{\pi(r_i^4 - r_o^4)G}$$

Output Torque $= (5.0)(15,159) = 75,795$ IN.-LB

$$\theta = \frac{(2)(75,795)(24)}{\pi\left((9.3125)^4 - (9.1325)^4\right)(11\times10^6)} = .000185 \text{ Radians}$$

$$\theta = \frac{(2)(75,795)(1.82 + 1.82)}{\pi\left((7.80)^4 - (7.065)^4\right)(11\times10^6)} = .000013 \text{ Radians}$$

$$\theta_{RG_{Total}} = .000185 + .000013 = \underline{.000198 \text{ Radians}}$$

$$\text{Stroke} = \frac{(.069417)(12.75)(2.15)}{(7.90)} = \underline{0.241}$$

(Referred to N_2)

SPLINES FP 500 AND 501

Input Spline $T = 3,939$ IN.-LB

21T
16/32 DP
30° Press ∢

$$K = \frac{T}{(FW)(PD)^2} = \frac{3,939}{(1.25)(1.3125)^2}$$

$K = 564$

Bearing Stress $= 1,127$ LB/sq in.

Sun Gear Spline $T = 3,939$ IN.-LB

22T
16/32 DP
30° Press ∢

$$K = \frac{3,939}{(1.09)(1.375)^2} = 1,911$$

Bearing Stress $= 3,822$ LB/sq in

Sun Planet Spline $T = 943$ IN.-LB

$$K = \frac{943}{(.50)(1.5625)^2} = 772.5$$

25T
16/32 DP
30° Press ∢

Bearing Stress = 1545 LB/sqin

Quill Shaft to N_3 Gear $T = 943$ IN.-LB

$$K = \frac{943}{(.60)(1.0)^2} = 1572$$

16T
16/32 DP
30° Press ∢

Bearing Stress = 3,144 LB/sqin

N_2 Gear to N_3 Gear $T = 3,911$ IN.-LB

$$K = \frac{3,911}{(.68)(2.0625)^2} = 1352$$

33T
16/32 DP
30° Press ∢

Bearing Stress = 2,704 LB/sqin

Output Gear Spline

146 T
8 DP
20° Press ∢

FW = .500"
P.D. = 18.25"

$T = 75,797$ IN.-LB

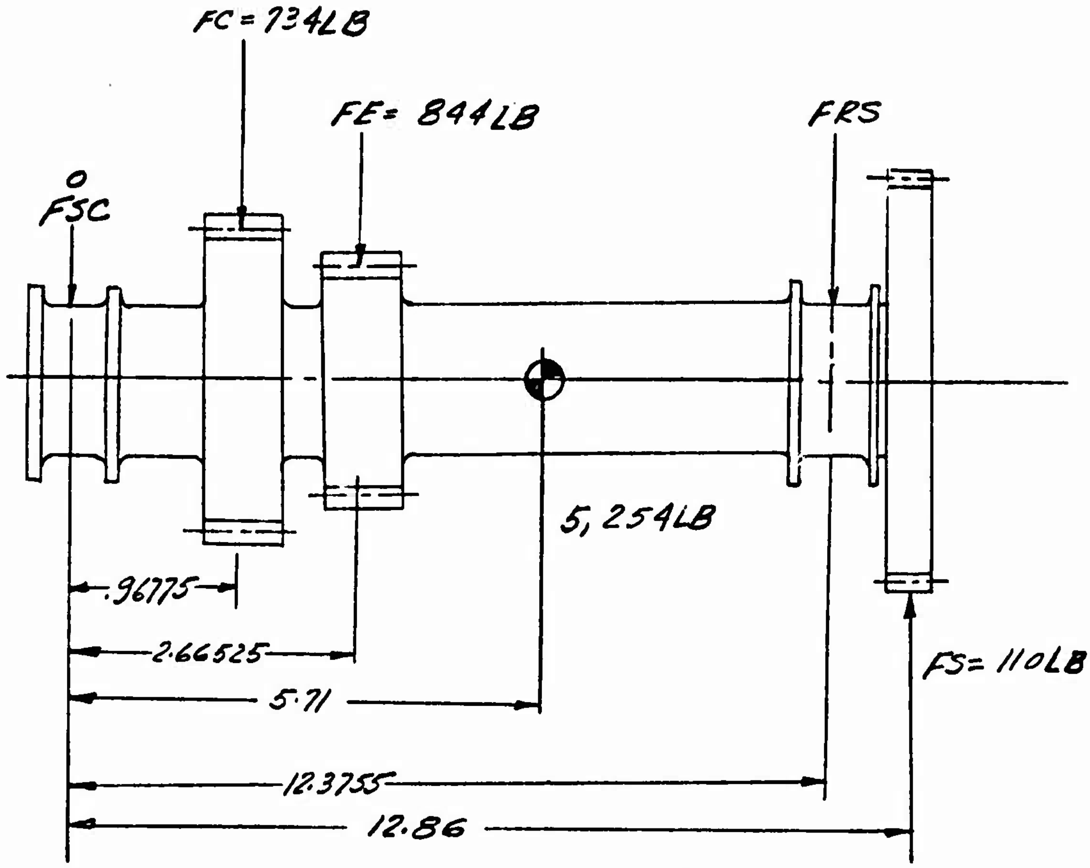
For .002 Misalignment

$FW \times PD^2 = 43.2536$

$$\text{Compressive Stress} = \frac{12,125.8}{\dfrac{0.5}{\left(\dfrac{43.2536}{(18.25)^2}\right)}} = \frac{(12,125.8)(43.2536)}{(0.5)(18.25)^2}$$

$$S_c = \underline{\underline{3,149 \text{ PSI.}}}$$

PLANET SUPPORT RINGS



$$Rev\ Planets = \frac{1}{1+\frac{N_4 N_2}{N_5 N_1}} = \frac{1}{1+\frac{(79)(158)}{(43)(66)}} = \frac{1}{1+4.398}$$

$$Rev\ Planets = 0.185248$$

$$Rev\ Planets\ (RPM) = (8000)(0.185248) = 1,482$$

$$CF = mr\omega^2 = \frac{14.659}{32.2}\left(\frac{5.75}{12}\right)\left[\frac{2\pi(1482)}{60}\right]^2$$

$$CF = 5,254\ LB$$

Design Torque & Design Speed

$$\Sigma M_0 = (734)(.96775) + (844)(2.66525) - (5254)(5.71)$$
$$-(110)(12.86) + (FRS)(12.3755) = 0$$

$$(12.3755)(FRS) = -710.3285 - 2{,}249.471 + 30{,}000.34 + 1{,}414.6000$$

$$FRS = \frac{28{,}455.1405}{12.3755} = 2{,}299.31\ LB \downarrow$$

$$\Sigma F_y = FSC + 734 + 844 - 5{,}254 + 2{,}299.31 - 110$$

$$FSC = 1486.69\ LB \downarrow$$

Design Torque & ORPM

$$\Sigma M_o = (734)(.96775) + (844)(2.66525) - 110(12.86) + (FRS)(12.3755) + (14.659)(5.71) = 0$$

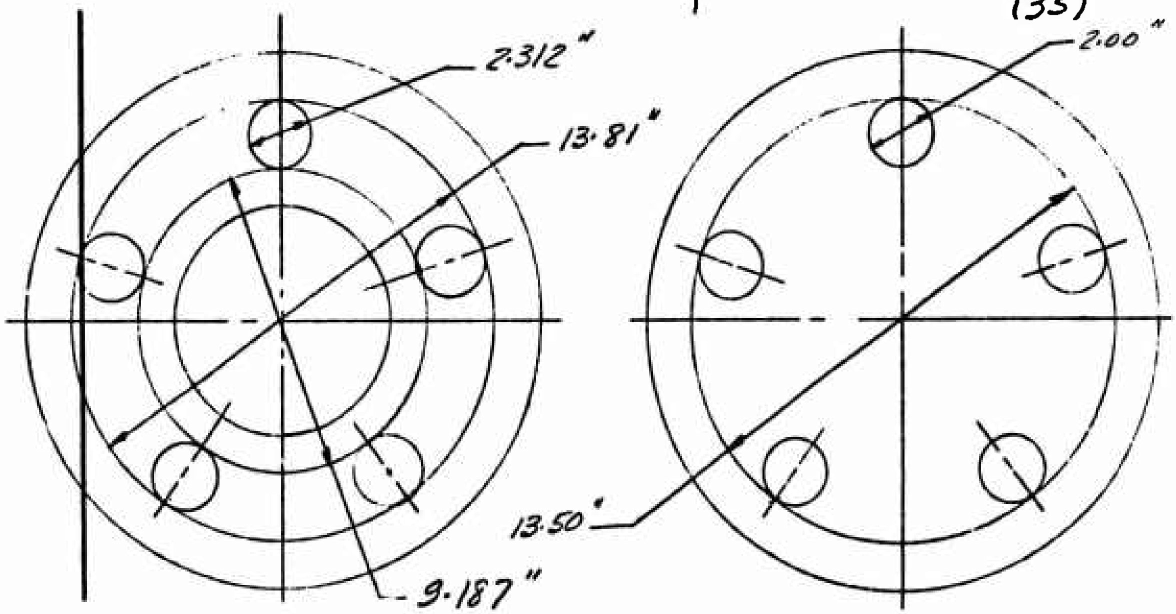
$$(FRS)(12.3755) = -710.3285 - 2{,}249.471 + 1414.6 - 83.70289$$

$$FRS = -\frac{1628.90239}{12.3755} = -131.62\ LB \uparrow$$

$$\Sigma F_y = 0 = 734 + 844 + 14.659 - 110 + FSC - 131.62$$

$$FSC = -1351.04\ LB \uparrow$$

(35)



Max Load

Inner Ring = 1351 LB

Outer Ring = 1487 LB

Max Load

Outer Ring = 2,299 LB

Outer Ring – 2nd Planet Gear Position

$$S_c = 0.591\sqrt{PE\left(\frac{D_1 - D_2}{D_1 D_2}\right)}$$

Width Ring = .52

Min Contact = .29

Max Contact = .372

$$S_c = 0.591\sqrt{\frac{(1487)}{.29}(30\times10^6)\left(\frac{11.498}{(13.81)(2.312)}\right)}$$

$$S_c = 0.591\sqrt{55{,}395.57}\times10^3$$

$$S_c = (0.591)(235.36)\times10^3 = \underline{139{,}099\ PSI}\ \text{Max}$$

$$= \underline{122{,}815\ PSI}\ \text{Min}$$

Inner Ring – 2nd Planet Gear Position

$$S_c = 0.591\sqrt{PE\frac{(D_1 + D_2)}{D_1 D_2}}$$

$$S_c = 0.591\sqrt{\frac{(1351)}{.29}\left(\frac{11.499}{(9.187)(2.312)}\right)(30\times10^6)}$$

$$S_c = 0.591\sqrt{75{,}661.88}\times10^3$$

$$S_c = (0.591)(275.067)\times10^3 = \underline{162{,}256\ PSI}\ \text{Max}$$

$$= \underline{143{,}261\ PSI}\ \text{Min}$$

Outer Ring – 1st Planet Gear Position

$$S_c = 0.591\sqrt{PE\left(\frac{D_1 - D_2}{D_1 D_2}\right)}$$

Width Ring = .60

Min Contact = .372

Max Contact = .454

$$S_c = 0.591\sqrt{\frac{(2{,}299)}{.372}(30\times10^6)\left(\frac{11.50}{(13.50)(2.0)}\right)}$$

$$S_c = 0.591\sqrt{78{,}968}\ \times 10^3$$

$$S_c = (0.591)(281.01)\times 10^3 = \underline{166{,}078\ PSI}\quad Max$$

$$(37)\qquad = \underline{150{,}333\ PSI}\quad Min$$

Outer Ring – 2nd Planet Gear Position

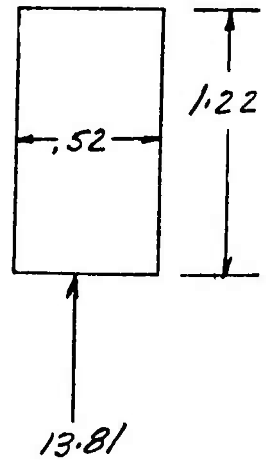
$$\theta = \frac{360}{10} = 36^\circ$$

$$\theta = 0.6283 \text{ Radians}$$

$$C = \cos\theta = 0.809016$$

$$S = \sin\theta = 0.587785$$

$$\cot\theta = 1.376381$$



$$I = \frac{bh^3}{12} = \frac{(.52)(1.22)^3}{12} = .078686 \text{ in.}^4$$

$$R = \left[\frac{13.81+1.22}{2}\right] = 7.515''$$

Outward Displacement – Radial

$$\sigma = \frac{WR^3}{2EI}\left[\frac{1}{S^2}\left(\frac{1}{2}\theta + \frac{1}{2}sc\right) - \frac{1}{\theta}\right]$$

$$\sigma = \frac{(1487)(7.515)^3}{(2)(30\times10^6)(.0787)}\left[\frac{1}{(.587785)^2}\left(\frac{.6283}{2} + \frac{(.809016)(.587785)}{2}\right) - \frac{1}{.6283}\right]$$

$$\sigma = (1336.509)(10^{-4})\left[2.894(.31415 + .237763) - 1.591596\right]$$

$$\sigma = (1336.509)(10^{-4})\left[.005640\right] = \underline{.000754''\ Outward}$$

INward Displacement — Radial

$$\sigma = \frac{WR^3}{4EI}\left(\frac{2}{\theta} - \frac{1}{S} - \theta\frac{C}{S^2}\right)$$

$$\sigma = \frac{(1336.509)(10^{-4})}{2}\left[\frac{2}{.6283} - \frac{1}{.587785} - .6283\left(\frac{.809016}{(.587785)^2}\right)\right]$$

$$\sigma = (668.25)(10^{-4})\left[3.183192 - 1.701302 - 1.471252\right]$$

$$\sigma = (668.25)(10^{-4})(.010638) = \underline{.00071088}\text{" Inward}$$

$$-M = -\frac{1}{2}WR\left(\frac{1}{\theta} - \cot\theta\right)$$

$$-M = -\frac{1}{2}(1487)(7.515)\left(\frac{1}{.6283} - 1.376381\right)$$

$$-M = -5,587.40(.215215) = -1202.49 \text{ IN.-LB}$$

$$S = -\frac{MC}{I} = -\frac{(1202.49)(.61)}{.078686} = \underline{-9,322 \text{ PSI}}$$

$$M = \frac{1}{2}WR\left(\frac{1}{S} - \frac{1}{\theta}\right) = 5,587.40\left(\frac{1}{.587785} - \frac{1}{.6283}\right)$$

$$M = 5,587.40(.109705) = 612.97 \text{ IN.-LB}$$

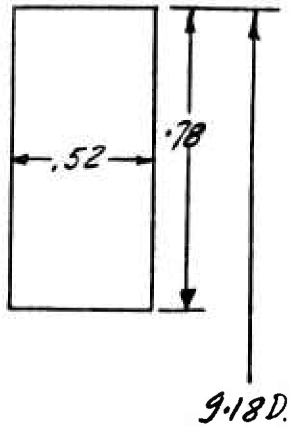
$$S = \frac{MC}{I} = \frac{(612.97)(.61)}{.078686} = \underline{4,752 \text{ P.S.I.}}$$

$$T = \frac{1}{2}W\left(\frac{1}{S}\right) = \frac{1}{2}(1487)\left(\frac{1}{.587785}\right) = 1,264.91 \text{ LB}$$

$$S = \frac{P}{A} = \frac{1264.91}{(1.22)(.52)} = \underline{1,994 \text{ PSI}}$$

Inner Ring – 2nd Planet Gear Position

Inward Displacement – Radial



$$I = \frac{bh^3}{12} = \frac{(.52)(.78)^3}{12} = .020563 \text{ IN.}^4$$

$$R = \left[\frac{9.18 - .78}{2}\right] = 4.20"$$

$$\delta = \frac{WR^3}{2EI}\left[\frac{1}{S^2}\left(\frac{\theta}{2} + \frac{SC}{2}\right) - \frac{1}{\theta}\right]$$

$$\delta = \frac{(351)(4.20)^3}{(2)(30 \times 10^6)(.020563)}\left[\frac{1}{(.587785)^2}\left(\frac{.6283}{2} + \frac{(.587785)(.809016)}{2}\right) - \frac{1}{.6283}\right]$$

$$\delta = 21,127 \times 10^{-6}\left[2.894429(.31415 + .237763) - 1.591596\right]$$

$$\delta = 81,127 \times 10^{-6}\left[.005876\right] = \underline{.000477"} \text{ inward}$$

Outward Displacement — Radial

$$\delta = \frac{WR^3}{4EI}\left[\frac{2}{\theta} - \frac{1}{S} - \theta\frac{C}{S^2}\right]$$

$$\delta = \frac{81,127\times10^{-6}}{2}\left[\frac{2}{.6283} - \frac{1}{.587785} - \frac{(.6283)(.809016)}{(.587785)^2}\right]$$

$$\delta = \frac{81,127\times10^{-6}}{2}(.010638) = \underline{.000432'' \text{ Outward}}$$

$$+M = \frac{1}{2}WR\left(\frac{1}{\theta} - \cot\theta\right)$$

$$M = \frac{1}{2}(1351)(4.20)(.215215) = 610.59 \text{ IN.-LB}$$

$$S = \frac{Mc}{I} = \frac{(610.59)(.39)}{.020563} = \underline{11,580 \text{ PSI}}$$

$$-M = -\frac{1}{2}WR\left(\frac{1}{S} - \frac{1}{\theta}\right)$$

$$-M = -\frac{1}{2}(1351)(4.20)(.109705) = -311.24 \text{ IN.-LB}$$

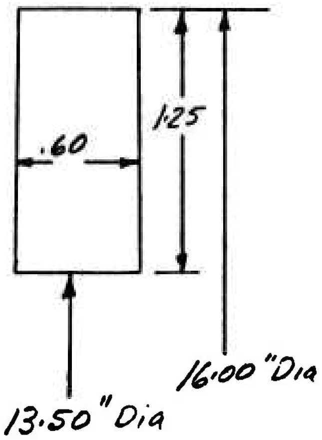
$$S = -\frac{Mc}{I} = -\frac{(311.24)(.39)}{.020563} = \underline{5,903 \text{ PSI}}$$

$$T = \frac{1}{2}W\left(\frac{1}{S}\right) = \frac{1}{2}(1351)\left(\frac{1}{.587785}\right) = 1,149.23 \text{ LB}$$

$$S = \frac{P}{A} = \frac{1,149.23}{(.78)(.52)} = \underline{2,833 \text{ PSI}}$$

Outer Ring - 1st Planet Gear Position

$$I = \frac{bh^3}{12} = \frac{(.60)(1.25)^3}{12} = .097656 \text{ in.}^4$$



$$R = \frac{(13.50 + 1.25)}{2} = 7.375''$$

Outward Displacement - Radial

$$\sigma = \frac{WR^3}{2EI}\left[\frac{1}{S^2}\left(\frac{1}{2}\theta + \frac{1}{2}SC\right) - \frac{1}{\theta}\right]$$

$$\sigma = \frac{(2,299)(7.375)^3}{(2)(30 \times 10^6)(.097656)}\left[.005640\right]$$

$$\sigma = .000888'' \text{ outward}$$

Inward Displacement - Radial

$$\sigma = \frac{WR^3}{4EI}\left(\frac{2}{\theta} - \frac{1}{S} - \frac{\theta C}{S^2}\right)$$

$$\sigma = \frac{(2\,299)(7.375)^3}{(4)(30 \times 10^6)(.097656)}(.010638)$$

$$\sigma = .000837'' \text{ Inward}$$

$$-M = -\frac{1}{2}WR\left(\frac{1}{\theta} - \cot\theta\right)$$

$$-M = -\frac{(2299)(7.375)}{2}(.215215) = -1824 \text{ IN.-LB}$$

$$S = -\frac{MC}{I} = -\frac{(1824)(.625)}{.097656} = \underline{11,673 \text{ PSI}}$$

$$M = \frac{1}{2} WR\left(\frac{1}{S} - \frac{1}{\theta}\right) = \frac{(2299)(7.375)}{2}(.109705)$$

$$M = 930.03 \text{ IN.-LB}$$

$$S = \frac{MC}{I} = \frac{(930.03)(.625)}{.097656} = \underline{5,952 \text{ PSI}}$$

$$T = \frac{1}{2} W\left(\frac{1}{S}\right) = \frac{1}{2}(2299)\left(\frac{1}{.587785}\right) = 1,956 \text{ LB}$$

$$S = \frac{P}{A} = \frac{1956}{(1.25)(.60)} = \underline{2,608 \text{ PSI}}$$

LIST OF SYMBOLS

A	Planet Designation
B	Planet Designation
C	Planet Designation
G	Planet Designation
S	Sun Gear
X	Internal Gear
Y	Internal Gear
Z	Internal Gear
a	Pitch radius "A" Planet, in.
b	Pitch radius "B" Planet, in.
c	Pitch radius "C" Planet, in.
g	Pitch radius "G" Planet, in.
s	Pitch radius "S" Sun Gear, in.
x	Pitch radius "X" Internal Gear, in.
y	Pitch radius "Y" Internal Gear, in.
z	Pitch radius "Z" Internal Gear, in.
e	Distance Planet to Planet, in.
σ	Distance Planet to Planet, in.
d	Distance Planet to Planet, in.
d(2)	Diameter of Input Stage Drive Cylinder, in.
D(2)	Diameter of Output Stage Drive Cylinder, in.
Q-(3)	Torque, in.-lb
F-(3)	Tangential Force, lb
FR	Radial Force, lb
FSR	Radial Force on Support Ring, lb

R	Reduction Ratio
R-(3)	Pitch Radius, in.
L-(3)	Planet Face Width, in.
LPC	Planet Face Width Center, in.
LPE	Planet Face Width End, in.